Exam 2 Calc 1 10/5/2018

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. State the formal definition of the derivative of a function $f(x)$.

2. Find an equation of the tangent line to the curve $y = 3x^2 - x^3$ at the point (1,2).

3. A table of values for f, g, f', and g' is given below.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	–16	2	4	10
2	6	9	2	4
3	8	2	–3	7

a) If $h(x) = f(x) \cdot g(x)$, what is $h'(3)$ and why?

b) If $h(x) = f(x) / g(x)$, what is $h'(2)$ and why?

c) If $h(x) = f(g(x))$, what is $h'(1)$ and why?

4. Use the definition of the derivative to show that the derivative of $f(x) = \sqrt{x}$ is $f'(x) = \dfrac{1}{2\sqrt{x}}$.

5. If a snowball melts so that its surface area decreases at a rate of 1 cm^2/min, find the rate at which the diameter decreases when the diameter is 12 cm.

6. State and prove the Product Rule for derivatives. Make it clear how you use any assumptions.

7. Bunny is a calculus student at Enormous State University, and she's having some trouble. Bunny says "*Ohmygod*. This Calculus stuff is so hard! I swear, it seems like they just *try* to make it impossible! Like, I mean, with the product rule thingie? Why does it even need to be that hard? I mean, I know we saw a proof and everything, but wouldn't it be right to just do the derivative of the first one and multiply by the derivative of the second one, instead of do it their complicated way? Why can't that be right too?"

   Help Bunny by explaining as clearly as possible how you know her simple approach does or does not work.

8. a) Write a linearization for $f(x) = \sqrt[4]{x}$ at $x = 16$.

b) Use the linearization from part a to approximate $\sqrt[4]{16.08}$

9. a) Show why the derivative of $f(x) = \tan x$ is $\sec^2 x$.

b) Suppose $L(x)$ is a function for which $L'(x) = 1/x$ (for values of x that aren't 0). Let $g(x) = L(\cos x)$. What's $g'(x)$?

10. a) Find the slope of the tangent line to the curve with equation $x^2 - 2xy + y^2 = 3$ at the point $\left(\sqrt{3}, 0\right)$.

b) The point $\left(\sqrt{3}, 0\right)$ is one of the x-intercepts of the curve. Find the other x-intercept and show that the tangent line there is parallel to the one in part a.

Extra Credit (5 points possible): State and prove the Quotient Rule for derivatives. Make it clear how you use any assumptions.