Exam 3 Calculus 3 11/30/2018

Each problem is worth 10 points. Show adequate justification for full credit. Please circle all answers and keep your work as legible as possible.

1. Parametrize and give bounds for a line segment beginning at (3,4) and ending at (5,0).

2. Compute $\int_C \left(x^2 + y^2\right) dx - x\,dy$ along the quarter circle (centered at the origin) from (1,0) to (0,1), followed by the line segment from (0,1) to the origin, followed by the line segment from the origin to (1,0).

3. Let $\mathbf{F}(x, y) = \langle 4xy^3 + 1, 6x^2y^2 \rangle$, and let C be the line segment from (1,1) to (3,2). Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$.

4. Let $\mathbf{F}(x, y) = \langle x^2 + y^2, -xy \rangle$, and let C be a quarter circle (centered at the origin) from (1,0) to (0,1). Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$.

5. Let $\mathbf{F}(x, y, z) = \langle x, y, z \rangle$, and let S be the surface of a sphere with outward orientation, centered at the origin, with radius 2. Evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$.

6. Prove that if $\mathbf{F}(x,y,z)$ is a vector field whose component functions have continuous second-order partial derivatives, then div(curl $\mathbf{F}$) = 0. Make it clear how the requirement that the partials be continuous is important.

7. Bunny is a calc 3 student at a large state university and she's having some trouble. Bunny says "Ohmygod, I totally hate multiple choice math test questions! There was this one, it said to figure out which could be the middle component of a vector field thingy to be able to use the fundamental thingy, right? And the vector field was, like, $2xy$ in the first component, and $2yz$ in the third component, but they didn't tell you the middle component, there were just a bunch of choices. So how can you possibly answer that? I tried just testing each of the answers, but I think I got confused."

   Explain as clearly as possible to Bunny how she could find the middle component of her vector field other than just be guessing, and why your approach works.

8. Let $\mathbf{F}(x, y, z) = \langle -y, x, -3 \rangle$, and let S be the portion of $z = x^2 + y^2$ below $z = 4$, with upward orientation. Evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$.

9. Let $\mathbf{F}(x, y, z) = x^2z^2\,\mathbf{i} + y^2z^2\,\mathbf{j} + xyz\,\mathbf{k}$, and let S be the part of the paraboloid $z = x^2 + y^2$ that lies inside the cylinder $x^2 + y^2 = 4$, oriented upward. Evaluate $\iint_S \operatorname{curl}\mathbf{F} \cdot d\mathbf{S}$.

10. Let $\mathbf{F}(x, y, z) = \langle xy, y^2 - 4, 0\rangle$, and let S be the rectangle with vertices (2,0,0), (–2,0,0), (–2,0,1), and (2,0,1), oriented in the direction of the positive y-axis. Evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$.

Extra Credit (5 points possible): Let $\mathbf{F}(x, y, z) = \langle xy, y^2 - 4, 0\rangle$, and let S be a sphere with radius 1 centered at $(0, b, 0)$. Evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$.