Exam 2b **Real Analysis 1** **11/2/18**

Each problem is worth 10 points. Show adequate justification for full credit. Don't panic.

1. State the definition of the derivative of a function $f(x)$ at $x = a$.

2. a) State the definition of a set E being closed.

b) State the definition of a set E being open.

3. State some version of L'Hôpital's Rule.

4. a) State the definition of a compact set.

b) State the Heine-Borel Theorem.

c) Give an example of an open cover for $(0, 2018)$ that has no finite subcover.

5. State and prove the Product Rule for Derivatives, making clear how your hypotheses are necessary.

6. Prove that the product of continuous functions is continuous.

7. State and prove the Boundedness Theorem.

8. State and prove Fermat's Theorem.

9. a) Prove or give a counterexample: If $f'(x) > g'(x)$ for all $x \in (a, b)$, then $f(x) > g(x)$ for all $x \in (a, b)$.

b) Prove or give a counterexample: If $f(x) > g(x)$ for all $x \in (a, b)$, then $f'(x) > g'(x)$ for all $x \in (a, b)$.

10. a) Give an example of a function $f : [0,1] \to \mathbb{R}$ which is continuous on [0,1] but for which there does not exist $c \in (0,1)$ for which $f'(c) = \frac{f(b)-f(a)}{b-a}$ or show why one can't exist.

b) Give an example of a function $f : [0,1] \to \mathbb{R}$ which is differentiable on (0,1) but for which there does not exist $c \in (0,1)$ for which $f'(c) = \frac{f(b)-f(a)}{b-a}$ or show why one can't exist.