Problem Set 9 **Real Analysis 1** **Due 12/3/18**

Four of these problems will be graded, with each problem worth 5 points. Clear and complete justification is required for full credit. You are welcome to discuss these problems with anyone and everyone, but must write up your own final submission without reference to any sources other than the textbook and instructor.

1. [Kosmala 6.2.11] Give an example of a function $f : [0,1] \to \mathbb{R}$ that is
   (a) bounded but $f \notin R[0,1]$
   (b) $f \in R[0,1]$ but not monotone.
   (c) $f \in R[0,1]$ but neither continuous nor monotone.
2. [Kosmala Theorem 6.3.1 (b)] Show that if $f \in R[a,b]$ and c is a real constant, then $cf \in R[a,b]$, and $\int_a^b cf = c\int_a^b f$.
3. [Kosmala 6.3.4 (b)] Give an example of two functions $f, g : [a,b] \to \mathbb{R}$ that are not Riemann integrable, but fg is.
4. [Kosmala 6.4.2] If $f : [a,b] \to \mathbb{R}$ is continuous, find an antiderivative for f.
5. Give an example of a function that is differentiable but whose derivative is not Riemann integrable.
6. Give an example of a function $f : [0,1] \to \mathbb{R}$ for which $f(x) \geq 0$ $\forall x \in [0,1]$ and $\int_0^1 f = 0$, but $f(x) \neq 0$ for some $x \in [0,1]$.