Exam 2 Calc 1 10/4/2019

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. State the formal definition of the derivative of a function f(x).

2. Find an equation of the tangent line to the curve $y = \cos x - \sin x$ at the point $(\pi, -1)$.

3. Prove the Sum Rule for Derivatives, that if f(x) and g(x) are two differentiable functions then (f+g)'(x) = f'(x) + g'(x).

x	f(x)	g(x)	f'(x)	g'(x)
1	-1	3	5	4
2	6	7	2	5
3	9	3	1	7

4. A table of values for f, g, f', and g' is given below.

a) If $h(x) = f(x) \cdot g(x)$, what is h'(3) and why?

b) If h(x) = f(x) / g(x), what is h'(2) and why?

c) If h(x) = f(g(x)), what is h'(1) and why?

5. Differentiate

a)
$$f(x) = x^3 \cos x$$

b)
$$g(x) = \frac{\cos x}{x^3}$$

c)
$$h(x) = \cos(x^3)$$

6. State and prove the Quotient Rule for derivatives. Make it clear how you use any assumptions.

7. Bunny is a calculus student at Enormous State University, and she's having some trouble. Bunny says "*Ohmygod*. This Calculus stuff is so unfair! I swear, it's literally the least fair thing ever. There was this one function, like the cube root, right, and like, the professor was talking about it right at the end of class because somebody asked a question, right? And the question was about the tangent line when it's zero, but like, somehow the calculator said error, right? So the professor said there *is* a tangent line, but it's not wrong that the calculator said error, which is totally contradictory and unfair, but class was ending so there were, like, 200 people standing up in front of me and I have no idea what he was saying, so now I'm going to *fail*!"

Help Bunny by explaining as clearly as possible why a calculator might get an error in connection with such a question, but the tangent line might still exist.

8. Use the definition of the derivative to find the derivative of $f(x) = \frac{1}{x}$.

9. At noon, ship A is 100 km west of ship B. Ship A is sailing south at 30 km/h and ship B is sailing west at 20 km/h. How fast (to the nearest tenth of a mile per hour) is the distance between them changing at 2:00 PM?

10. a) The curve with implicit equation $x^4 + y^4 = 2$ is sometimes called a squircle (like a mix of a square and a circle). Find the slope of the tangent line to the squircle at the point (1, -1).



b) The curve with implicit equation $x^4 + 7xy^2 + 2y^4 = 2$ is hereby called a blorg. What is the slope of the line tangent to the blorg at (0,1)?



Extra Credit (5 points possible): Find the coordinates of the highest and lowest points on the blorg from #10.