Exam 3 Calc 1 10/23/2019

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. What is $(e^x)'$?

2. a) What is $(10^x)'$?

   b) What is $(\log_2 x)'$?

3. Evaluate $\lim_{x\to\infty} \frac{x}{e^x}$. Be sure to provide good justifications for your steps.

4. a) If $f(x) = x^3 \ln x$, what is $f'(x)$?

b) If $g(x) = \ln (x^3)$, what is $g'(x)$?

5. A 17 foot ladder is leaning against a wall. If the top slips down the wall at a rate of 4 ft/s, how fast will the foot be moving away from the wall when the top is 12 feet above the ground?

6. Show why the derivative of arctan x is what it is.

7. Biff is a calculus student at Enormous State University, and he's having some trouble. Biff says "Geez, calculus is hard! I though I had pretty much the perfect plan, you know? I've got, you know, one of those calculators that does derivatives for you, right? So I thought the exam would be really easy, but it totally didn't work. There was this one batch of questions on the exam with all these functions to do derivatives of, right, but it was the log of 7, and then like the inverse sine of one half, and *e* to the seven or something, right? But the calculator said it was zero for all of them, which is totally stupid since they're not even the same function, so they can't have the same derivative, right?"

   Explain clearly to Biff what's going on.

8. A research sample of 1000 zombie-spawning bacteria escapes the lab and begins to grow at a rate proportional to its population. After 5 hours there are 3000 bacteria. Give a formula for the number of bacteria after t hours.

9. A freshly brewed cup of coffee sits on a counter in a 21°C room at 95°C. After 5 minutes its temperature is down to 85°C. How soon will it be 60°C? [Hint: the general solution to Newton's Law of Cooling is $T(t) = T_s + Ae^{kt}$, where T_s is the temperature of the surrounding medium.]

10. Evaluate $\lim_{x\to\infty} \dfrac{\ln\left(1+\dfrac{1}{x}\right)}{\dfrac{1}{x}}$. Be sure to provide good justifications for your steps.

Extra Credit (5 points possible): Evaluate $\lim_{x\to\infty} \ln\left(1+\dfrac{1}{x}\right)^x$ and $e^{\lim_{x\to\infty} \ln\left(1+\frac{1}{x}\right)^x}$. Be sure to provide good justifications for your steps.