Examlet 1 **Foundations of Advanced Math** **10/12/19**

1. The sum of two odd integers is even.

Let x, y be odd

$x = 2a + 1$
$y = 2b + 1$ where $a, b \in \mathbb{Z}$

$x + y = 2a + 2b + 2$

$x + y = 2(a + b + 1)$

$a + b + 1$ is an integer by colsure

$\therefore$ $x + y$ is even by definition

Excellent

2. If $a \equiv_n b$ and $b \equiv_n c$, then $a \equiv_n c$.

$a \equiv_n b \iff n \mid b - a \iff b - a = nx$

where x, y are $\in \mathbb{Z}$

$b \equiv_n c \iff n \mid c - b \iff c - b = ny$

if we add nx and ny together,

$((c - b) + (b - a) = nx + ny$

$((c - a) = n(x + y)$ where $x + y$ is an integer by closure

$\therefore$ $((c - a) = n(x + y) \iff n \mid c - a \iff a \equiv_n c$ by definition

Nice.

3. Determine whether the statements $P \Rightarrow (Q \vee R)$ and $(P \Rightarrow Q) \vee (P \Rightarrow R)$ are logically equivalent.

P	Q	R	$Q \vee R$	$P \Rightarrow Q$	$P \Rightarrow R$	$P \Rightarrow (Q \vee R)$ *	$(P \Rightarrow Q) \vee (P \Rightarrow R)$ *
T	T	T	T	T	T	T	T
T	T	F	T	T	F	T	T
T	F	T	T	F	T	T	T
T	F	F	F	F	F	F	F
F	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T
F	F	T	T	T	T	T	T
F	F	F	F	T	T	T	T

Since the columns marked with * are the same in all conditions, $P \Rightarrow (Q \vee R)$ and $(P \Rightarrow Q) \vee (P \Rightarrow R)$ are logically equivalent. □

Great

4. There is no smallest positive real number.

Well, s'pose there <u>was</u> a smallest positive real number, and call it x, so $x > 0$. But multiplying both sides by $\frac{1}{2}$ gives us $\frac{x}{2} > 0$, so $\frac{x}{2}$ is also a positive real number. Also I'm pretty sure $2 > 1$, and multiplying both sides by x (since it's greater than 0) gives $2x > 1x$, and multiplying by $\frac{1}{2}$ now gives $x > \frac{x}{2}$, so $\frac{x}{2}$ is a positive real smaller than x, a contradiction. $\square$

induction

5. For any $n \in \mathbb{Z}^+$,

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

Suppose $n=1$. Then $\sum_{i=1}^{1} i = \frac{1\cdot(2)}{2} = 1$, making the statement true for $n=1$. Suppose the statement is true for some integer K. So $\sum_{i=1}^{K} i = \frac{k(k+1)}{2}$.

So $\sum_{i=1}^{K} i + (k+1) = \frac{k(k+1)}{2} + (k+1)$

$2\left(\sum_{i=1}^{K} i + (k+1)\right) = k(k+1) + 2(k+1)$

factoring,

$2\left(\sum_{i=1}^{K} i + (k+1)\right) = (k+2)(k+1)$

$\sum_{i=1}^{K} i + (k+1) = \frac{((k+1)+1)(k+1)}{2}$

$\sum_{i=1}^{K+1} i = \frac{(k+1)((k+1)+1)}{2}$,

Good.

making the statement true for $(k+1)$. So by induction, for any $n \in \mathbb{Z}^+$, $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$. □

$\sum_{i=1}^{K+1} i = \frac{(k+1)((k+1)+1)}{2}$

↑

goal