Problem Set 5 **Foundations** **Due 10/16/19**

Five of these problems will be graded (our choice, not yours!), with each problem worth 4 points. Clear and complete justification is required for full credit. You are welcome to discuss these problems with anyone and everyone, but must write up your own final submission without reference to any sources other than the textbook and instructor.

1. If $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ are odd functions with $g(x) \neq 0$, $\forall x \in \mathbb{R}$, then f/g is an odd function.

2. If $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ are even, then $f - g$ is even.

3. If $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ are decreasing, then $f \cdot g$ is decreasing.

4. If $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ are strictly increasing, then $f + g$ is strictly increasing.

5. If $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ have positive derivatives $\forall x \in \mathbb{R}$, then $f + g$ is increasing.

6. Let f and g be bounded functions, both with domain D. Then $f + g$ is a bounded function.

7. Let f and g be bounded functions, both with domain D. Then $f - g$ is a bounded function.

8. Let f and g be bounded functions, both with domain D. Then $f \cdot g$ is a bounded function.

9. Let f and g be bounded functions, both with domain D. Then f/g is a bounded function.

10. Critique the following proof of the proposition "The product of two odd functions, both with domain D, is odd":
    Let f and g be two odd functions, both with domain D, and consider their product $f \cdot g$. Then,

    $$(f \cdot g)(-x) = f(-x) \cdot g(-x) = -f(x) \cdot (-g(x)) = -(f(x) \cdot g(x)) = -(f \cdot g)(x).$$

    Therefore, $f \cdot g$ is odd.