Exam 2 **Modern Algebra 1** **10/9/20**

Do questions 1 through 7 and pick three of the remaining (lettered) questions for grading (check boxes of the lettered problems you want graded or I roll dice). Each problem is worth 10 points. Show good justification for full credit. Don't panic.

1. State the Fundamental Theorem of Arithmetic.

2. State the Division Algorithm.

3. Show that for a positive integer n, congruence modulo n is an equivalence relation on the set of integers.

4. State Lagrange's Theorem.

5. Let G be a group and H a subgroup of G. Define a relation $\sim$ on G by

$$a \sim b \iff ab^{-1} \in H.$$

Show that $\sim$ is an equivalence relation on G.

6. Find the subgroup of $\mathbb{Z}_{12}$ generated by [8].

7. Use the Euclidean Algorithm to find the greatest common divisor of 501 and 120.

□ A. Express the greatest common divisor of 501 and 120 as a linear combination of 501 and 120.

□ B. Which elements of $\mathbb{Z}_{20}$ are generators for the group?

□ C. In S_3, find all right cosets of $\langle(2\ \ 3)\rangle$.

□ D. Determine whether $\mathbb{Z}_2 \times \mathbb{Z}_4$ is cyclic.

□ E. The subgroup $A_4 = \langle (1\ \ 2\ \ 3), (1\ \ 2)(3\ \ 4) \rangle$ of S_4 has order 12. Does it have a subgroup of order 6?

□ F. Solve the equation $(1\ \ 3\ \ 2) \circ x \circ (1\ \ 4) = (1\ \ 5\ \ 2)$, that is, find an element x of S_5 that makes the equation true.