Exam 3 Modern Algebra 1 10/30/20

Do questions 1 through 7 and pick three of the remaining (lettered) questions for grading (check boxes of the lettered problems you want graded or I roll dice). Each problem is worth 10 points. Show good justification for full credit. Don't panic.

1. (a) State the definition of a ring.

   (b) Give an example of a ring without unity.

2. State the definition of a group isomorphism.

3. State the definition of the quotient group G/N.

4. (a) Determine whether $\langle (1\ 2\ 3) \rangle$ is a normal subgroup of S_3.

(b) Determine whether $\langle (1\ 2) \rangle$ is a normal subgroup of S_3.

5. Which elements of $\mathbb{Z}_{12}$ are zero divisors and why?

6. Prove that if $\theta : G \to H$ is a homomorphism, then $\ker \theta$ is a subgroup of G.

7. Prove that if $\theta : G \to H$ is a homomorphism, then $\ker \theta$ is a normal subgroup of G.

□ A. Show that if G is a group and N a normal subgroup of G, then the set G/N with operation as defined in question 3 forms a group.

□ B. Give an example of an integral domain which is not finite.

□ C. Prove that if R is a ring and 0 the additive identity of R, then for any $a \in R$, $0 \cdot a = a \cdot 0 = 0$.

□ D. Prove Lila's Theorem, that any subgroup of an Abelian group is normal.

□ E. Show that if G and H are isomorphic groups with G being Abelian, then H must also be Abelian.

□ F. Determine, with proof, whether the set of 2 by 2 matrices of the form

$$\begin{bmatrix} a & 0 \\ b & c \end{bmatrix}$$

with a, b, c real numbers, forms a ring with matrix addition and multiplication as the operations.