Exam 1 Calc 1 9/14/2020

Each problem is worth 10 points. For full credit provide complete justification for your answers.

Use the graph of $g(x)$ at the bottom of the page for problems 1 and 2:

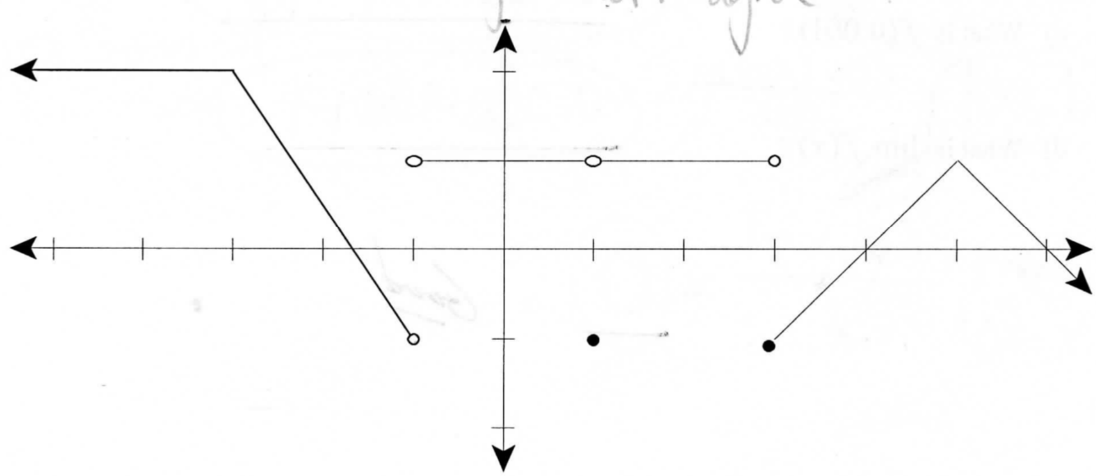
1. Find the following limits:
   a) $\lim_{x\to -1^-} g(x) = \underline{-1}$
   b) $\lim_{x\to -1^+} g(x) = \underline{1}$
   c) $\lim_{x\to -1} g(x) = \underline{\text{DNE}}$, limits from left & right don't agree
   d) $\lim_{x\to 1^+} g(x) = \underline{1}$
   e) $\lim_{x\to 1^-} g(x) = \underline{1}$
   f) $\lim_{x\to 1} g(x) = \underline{1}$, limit from left and right agree

Great

2. For which values of x does the function fail to be continuous?

-1 because the limits from left + right don't agree, 1 b/c the value of $f(1) \neq \lim_{x\to 1}$, and 3 b/c the limits from left + right don't agree

Excellent!



Booooom!

3. Evaluate $\lim_{x\to 2}\frac{(\cancel{x-2})(x-1)}{\cancel{x-2}}$. $= \lim_{x\to 2}(x-1)$

Pow!

$(2-1)$

$\boxed{=1}$

Great

4. Let $f(x)=\frac{\cos x-1}{x}$. ***Make sure your calculator is in radian mode.*** Give answers accurate to at least 8 decimal places. -> eh I'll do 10 decimal for best accuracy.

a) What is $f(0.1)$? -> $\frac{\cos(.1)-1}{.1} = -0.0499583472$

b) What is $f(0.01)$? -> $\frac{\cos(.01)-1}{.01} = -0.0049999583$

c) What is $f(0.001)$? -> $\frac{\cos(.001)-1}{.001} = -0.0004999999$

$4.999999 E^{-4}$

d) What is $\lim_{x\to 0^+} f(x)$?

The closer that f(x) gets to 0, the function seems to be getting closer and closer to 0 as well, according to my obserbations. When I plug .0000001 to the equation, I got 0 as an answer but you know calculators.

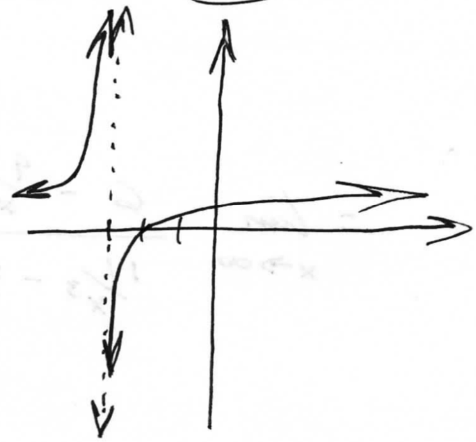
Yes!

5. Find the limits:

a) $\lim\limits_{x \to -3^+} \dfrac{x+2}{x+3}$

x	$f(x)$
-2.9	-9
-2.99	-99
-2.999	-999

Looks like $-\infty$



b) $\lim\limits_{x \to -3^-} \dfrac{x+2}{x+3}$

x	$f(x)$
-3.1	11
-3.01	101

looks like $+\infty$

6. Evaluate $\lim_{x\to\infty} \dfrac{6x^3 - 9x^2 - 6x}{11 - 5x - 2x^3}$.

$$\lim_{x\to\infty} \frac{6x^3 - 9x^2 - 6x}{11 - 5x - 2x^3} \cdot \frac{\frac{1}{x^3}}{\frac{1}{x^3}}$$

$$\lim_{x\to\infty} \frac{\frac{6x^3}{x^3} - \frac{9x^2}{x^3} - \frac{6x}{x^3}}{\frac{11}{x^3} - \frac{5x}{x^3} - \frac{2x^3}{x^3}}$$

7

$$\lim_{x\to\infty} \frac{6 - \frac{9}{x} - \frac{6}{x^2}}{\frac{11}{x^3} - \frac{5}{x^2} - 2}$$

$$\lim_{x\to\infty} \frac{6 - \cancel{\frac{9}{x}}^{\,\to 0} - \cancel{\frac{6}{x^2}}^{\,\to 0}}{\cancel{\frac{11}{x^3}}^{\,\to 0} - \cancel{\frac{5}{x^2}}^{\,\to 0} - 2}$$

Excellent!

$$\lim_{x\to\infty} \frac{6}{-2} = -3$$

$$* \quad \lim_{x\to\infty} \frac{6(1{,}000{,}000)^3 - 9(1{,}000{,}000)^2 - 6(1000{,}000)}{11 - 5(1{,}000{,}000) - 2(1{,}000{,}000)^3} = -2.9999955$$

Nice double-check.

7. Biff is a calculus student at Enormous State University, and he's having some trouble. Biff says "Well, crap. Calc is totally killing me. I thought it would be easy because I've got a cool graphing calculator, right? But I guess it's broken or something. There was this question on our test where the professor said it's not continuous, like there's a hole, but when I graphed *x* squared minus 4 over *x* minus 2, it totally shows no hole. So I'm pretty sure the professor is an idiot, because the calculator definitely knows how to graph, right?"

Help Biff by explaining as clearly as you can whether there's a hole in his function, where, and why.

$$\boxed{\frac{x^2-4}{x-2}}$$

There is a hole in Biff's function at x=2. When x=2, the denominator of the function is equal to 0, meaning the y value for that x input does not exist. The graphing calculator does not show the hole in the function because the limit for that equation as x approaches 2 does exist, and every value infinitely close to x=2 does exist, it only does not exist at exactly x=2.

Good.

8. An extremely shiny stainless steel cylinder falls toward the surface of Mars after an engine malfunction shortly before planned landing so that the height of the cylinder above ground level is given by is given by $h(t) = -1.86t^2 + 7.44$ for values of t between 0 and 2.

Give answers accurate to at least 8 decimal places.

a) Find the average velocity of the cylinder over the interval [1.5, 2].

$$\frac{3.255 - 0}{1.5 - 2} = \frac{3.255}{-.5} = -6.51$$

b) Find the average velocity of the cylinder over the interval [1.9, 2].

$$\frac{0.7254 - 0}{1.9 - 2} = \frac{0.7254}{-0.1} = -7.254$$

c) Find the average velocity of the cylinder over the interval [1.99, 2].

$$\frac{0.074214 - 0}{1.99 - 0} = \frac{0.074214}{-0.01} = -7.4214$$

d) Estimate the cylinder's instantaneous velocity at $t = 2$.

$1.999 \rightarrow -7.438$

$1.9999 \rightarrow -7.44$ m/s

9. For the function $f(x)=\dfrac{(x-3)(x+1)}{|x-3|}$, evaluate the following limits and explain your reasoning clearly:

a) $\lim\limits_{x\to 3^-} f(x) = -4$

b) $\lim\limits_{x\to 3^+} f(x) = 4$

c) $\lim\limits_{x\to 3} f(x)$ does not exist, because $\lim\limits_{x\to 3^-}$ and $\lim\limits_{x\to 3^+}$ do not agree

x	y
2.5	-3.5
2.9	-3.9
2.99	-3.99
3.5	4.5
3.1	4.1
3.01	4.01

$$\frac{(2.5-3)(2.5+1)}{|2.5-3|} = \frac{(-0.5)(3.5)}{0.5} = -3.5$$

$$\frac{(-0.1)(3.9)}{0.1} = -3.9$$

$$\frac{(0.5)(4.5)}{0.5} = 4.5 \qquad \frac{(0.1)(4.1)}{0.1} = 4.1$$

Good

10. A function of the form $f(x) = \begin{cases} 0 & \text{for } x \le 0 \\ 0.1x & \text{for } 0 < x \le 1 \\ c & \text{for } 1 < x \le 2 \\ -0.1x + 0.3 & \text{for } 2 < x \le 3 \\ 0 & \text{for } 3 < x \end{cases}$ is meant to be continuous.

What value does c need to have, and why?

The value of c has to be equal to .1 for it to be continuous both the low end of the equasion $-.1x+.3$ and the high end of 0.1x are equal to 0.1 so the equasion would just be $f(x)=.1$ when $1<x\le2$

$f(2)=-.1x+.3=.1$

$f(1)=.1x=.1$

Exactly!