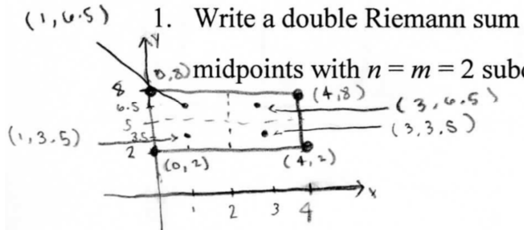
Exam 2 Calc 3 10/19/2020

Each problem is worth 10 points. For full credit provide complete justification for your answers. All integrals should be set up in terms of a single coordinate system, i.e., if you use cylindrical your integral should involve no x or y, etc.

1. Write a double Riemann sum for $\iint_R f\,dA$, where $R = \{(x, y) : 0 \le x \le 4, 2 \le y \le 8\}$ using midpoints with $n = m = 2$ subdivisions.



$\Delta X = 2$
$\Delta Y = 3$
$\Delta A = 3 \cdot 2 = 6$

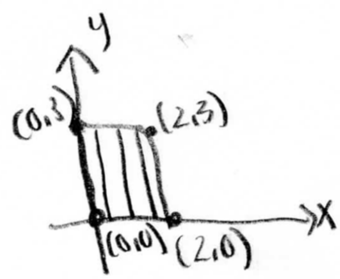
Good

$$\boxed{6[f(1,3.5) + f(1,6.5) + f(3,3.5) + f(3,6.5)]}$$

10

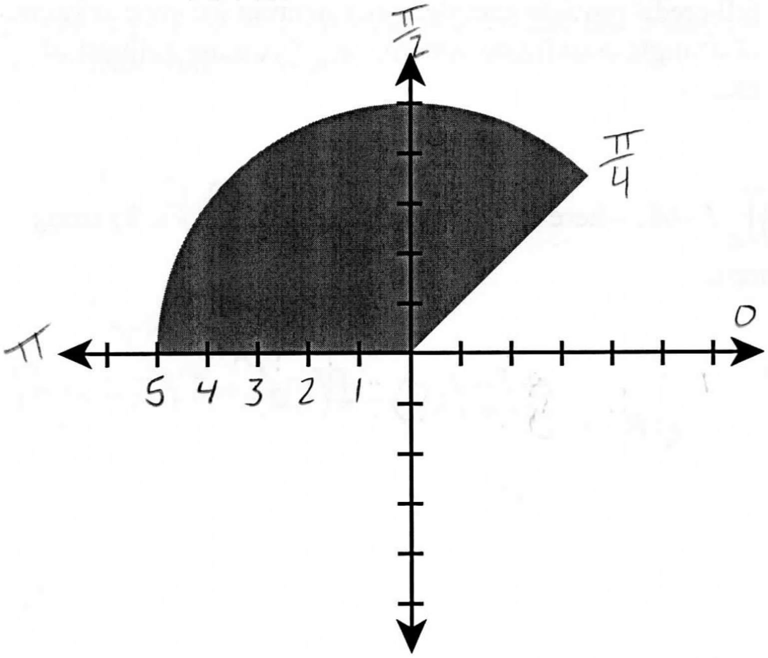
2. Let $f(x,y) = 40 - x + 2y$. Let R be the rectangle with vertices (0,0), (2,0), (2,3), and (0,3). Set up an iterated integral for $\iint_R f\,dA$.

$$\int_0^2 \int_0^3 40 - x + 2y \, dy\,dx$$



Good

3. Set up an iterated integral for the total mass of a plate shaped like the region shown below, with density $\rho(x, y) = kx$.



Excellent

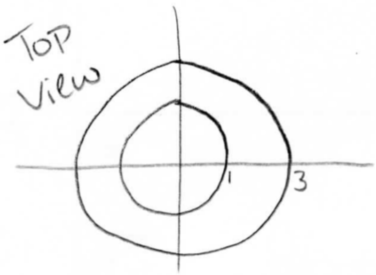
change

$$\int_{\frac{\pi}{4}}^{\pi} \int_0^5 Kxr\,dr\,d\theta$$

$$\int_{\frac{\pi}{4}}^{\pi} \int_0^5 (K)(r\cos\theta)(r)\,dr\,d\theta$$

integrand 1

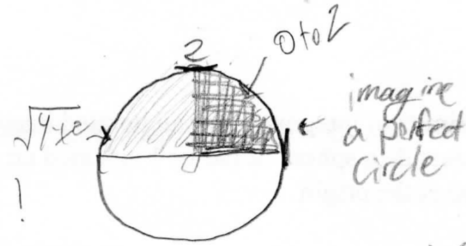
4. Set up limits of integration for an iterated integral for the volume of the region above the *xy*-plane, outside a sphere of radius 1 centered an the origin, and inside a sphere of radius 3 centered at the origin.



$$\int_0^{2\pi}\int_0^{\frac{\pi}{2}}\int_1^{3} (\rho^2 \sin\phi)\, d\rho\, d\phi\, d\theta$$

Good

5. Evaluate $\int_0^2 \int_0^{\sqrt{4-x^2}} \sqrt{x^2+y^2}\, dy\, dx$.



Switch to Polar!!

$\sqrt{x^2+y^2} = r$ || $dy\,dx = r\,dr\,d\theta$

r from 0 to 2 θ from 0 to $\pi/2$

Excellent!

$$\int_0^{\frac{\pi}{2}} \int_0^2 r^2\, dr\, d\theta$$

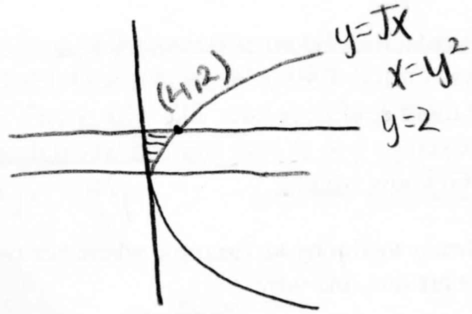
$$\frac{1}{3} r^3 \Big|_0^2 = \frac{8}{3}$$

$$\int_0^{\pi/2} \frac{8}{3}\, d\theta = \frac{8}{3} \cdot \frac{\pi}{2} = \frac{8\pi}{6} = \frac{4\pi}{3}$$

$y=2$

$y=\sqrt{x}$

6. Evaluate $\int_0^4 \int_{\sqrt{x}}^2 \frac{1}{y^3+1}\,dy\,dx$.



Change order of integration

$$\int_0^2 \int_0^{y^2} \frac{1}{y^3+1}\,dx\,dy$$

$$\int_0^2 \frac{x}{y^3+1}\Big|_0^{y^2} dy$$

$$\int_0^2 \frac{y^2}{y^3+1}\,dy$$

Let $u=y^3+1$

$du=3y^2$

$\frac{1}{3}du=y^2$

2^3+1

$$\frac{1}{3}\int_1^9 \frac{1}{u}\,du$$

0^3+1

$$\frac{1}{3}\left(\ln u\Big|_1^9\right)$$

$$\frac{1}{3}\ln 9-\frac{1}{3}\ln 1$$

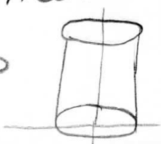
$$\boxed{\frac{1}{3}\ln 9}$$

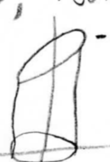
Great

7. Bunny is a calculus student at Enormous State University, and she's having some trouble. Bunny says "OMG! Calc 3 is just so much! It's like, these things I thought I understood before are more confusing now, like with polar? So like sometimes you go zero to 2 pi, but sometimes you go 0 to pi over two and take it times four, but sometimes you can't do that. How do you know which?"

Explain clearly to Bunny an example where her two ways are equal, and also an example where they are not, and why.

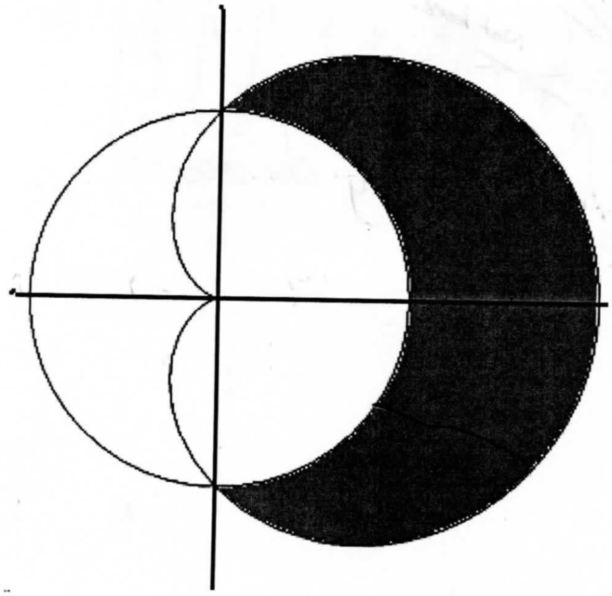
To determine if these two will be equal, you need to think about symmetry.

For example, something like a cylinder with a flat top  centered about the origin is going to be the exact same in all quadrants, and therefore you could just solve for one and multiply by 4.

On the other hand, something with a slanted top such as  will not be equal in all quadrants because the height is not uniform. You therefore would need to use all the way from 0 to 2π to account for all changes.

Excellent

8. Set up an iterated integral for the area the area of the region bounded on the inside by the circle of radius 4 and on the outside by the cardioid $r = 4(1 + \cos(\theta))$



$$\boxed{\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_{4}^{4(1+\cos\theta)} 1 \, r \, dr \, d\theta}$$

Great

$r = 4$

$r = 4(1 + \cos\theta)$

$4 = 4(1 + \cos\theta)$

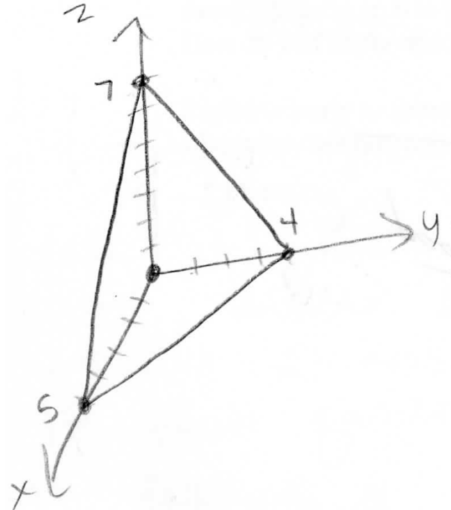
$1 = 1 + \cos\theta$

$0 = \cos\theta$

$\theta = -\frac{\pi}{2}, \frac{\pi}{2}$

9. Set up limits for a triple integral $\iiint_E xy\,dV$ where E is the solid tetrahedron with vertices (0,0,0), (5,0,0), (0,4,0), (0,0,7).

$$= \int_{x=0}^{x=5}\int_{y=0}^{y=-\frac{4}{5}x+4}\int_{z=0}^{z=7-7/5x-7/4y} xy\,dz\,dy\,dx$$



$z=7-\frac{7}{5}x-\frac{7}{4}y$

$y=-\frac{4}{5}x+4$

Excellent

10. Set up an iterated integral for the volume of the solid that lies within the sphere $x^2 + y^2 + z^2 = 4$, above the xy-plane, and outside the cone $z = 2\sqrt{x^2 + y^2}$.

$$\int_0^{2\pi} \int_{\arctan \frac{1}{2}}^{\frac{\pi}{2}} \int_0^{2} 1\, \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$