(Easier) Practice Quiz 2   Calc 3   11/2/20

1. Compute $\int_C \left\langle y^2, xy \right\rangle \cdot d\vec{r}$ for a path C given by $\vec{r}(t) = \left\langle 2+3t, 1-5t \right\rangle$ for $0 \le t \le 1$.

We can try using the Fun. Thrm. for Line Integrals, hoping for that easy way, but it doesn't apply here because there's no potential function (we can confirm this by noting that the partial of the coefficient of i with respect to y is $2y$, which is different from the y we get as the partial with respect to x of the coefficient of j).

So we go through the usual steps for computing a line integral. The path is already parametrized for us. $\mathbf{F}(\mathbf{r}(t)) = <(1-5t)^2, (2+3t)(1-5t)>$, and $\mathbf{r}'(t) = <3, -5>$. Then the integral we're asked about works out to 45.5.

(Harder) Practice Quiz 2   Calc 3   11/2/20

1. Compute $\int_C \vec{F} \cdot d\vec{r}$ for the vector field $\vec{F}(x, y) = x^2 y\,\vec{i} + y^3\,\vec{j}$ and with C an arc of a circle (centered at the origin) of radius 3 passing counterclockwise through the first and second quadrants.

The mixed partials show that no potential function exists, so we have to do it the hard way.

Our path parametrizes as $x(t) = 3\cos t$, $y(t) = 3\sin t$, for values of t between 0 and π.

We work out $\mathbf{F}(\mathbf{r}(t)) = \langle (3\cos t)^2(3\sin t), (3\sin t)^3 \rangle$, and $\mathbf{r}'(t) = \langle -3\sin t, 3\cos t \rangle$.

Our integral becomes $\int_0^\pi \langle 27\cos^2 t\sin t, 27\sin^3 t\rangle \cdot \langle -3\sin t, 3\cos t\rangle\, dt$ or

$\int_0^\pi \left(-81\cos^2 t\sin^2 t + 81\sin^3 t\cos t\right) dt$. This should work out, by way of a simple $u = \sin t$ substitution on the second part and some harder double and half angle identities applied to the first part, to -81/8.