Exam 1 **Real Analysis 1** **9/30/20**

Do questions 1 through 7 and pick three of the remaining (lettered) questions for grading (check boxes of the lettered problems you want graded or I roll dice). Each problem is worth 10 points. Show good justification for full credit. Don't panic.

1. State the definition of a sequence $\{a_n\}$ converging to a limit A..

2. State the definition of s_0 being an accumulation point of a set S.

3. Show that the limit of a function as x approaches a, if it exists, is unique.

4. Suppose that $\lim_{n\to\infty} a_n = +\infty$. Show that $\lim_{n\to\infty} \frac{1}{a_n} = 0$.

5. If $\{a_n\}$ is a Cauchy sequence and $S = \{a_n | n \in \mathbb{N}\}$ is finite, then $\{a_n\}$ is constant from some point on.

6. State and prove the Bolzano-Weierstrass Theorem for Sets.

7. Show that if $\lim_{x\to\infty} f(x) = A$ and $\lim_{x\to\infty} g(x) = B$, then $\lim_{x\to\infty} f \cdot g(x) = AB$

□ A. State and prove the Monotone Convergence Theorem.

☐ B. Give an example of a sequence that does not converge, but which has a subsequence which converges.

□ C. Show, directly from the definition, that $\lim_{x\to 3} x^2 = 9$.

□ D. (a) Prove or give a counterexample: If f is an odd function, then $\lim_{x \to 0} f(x) = 0$.

(b) Prove or give a counterexample: If f is an odd function with domain $\mathbb{R}$, then $\lim_{x \to 0} f(x) = f(0)$.

□ E. (a) Give an example of a sequence which is increasing and convergent.

(b) Give an example of a sequence which is increasing but not convergent.

(c) Give an example of a sequence which is bounded but not convergent.

□ F. Let s_0 be an accumulation point of S. Show that any neighborhood of s_0 contains infinitely many points of S.