Exam 3   Calc 1   10/22/2021

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. a) Let $f(x) = e^x$. What is $f'(x)$?

   b) Let $g(x) = \ln x$. What is $g'(x)$?

2. a) What is $(\arcsin x)'$?

   b) What is $(\arctan x)'$?

3. What is $\lim_{x \to 1} \dfrac{x^2 - 1}{x^2 - x}$?

4. A table of values for f, g, f', and g' is given below.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	–1	2	5	4
2	6	7	2	5
3	9	3	1	7

a) If $h(x) = \arcsin(f(x))$, what is $h'(2)$ and why?

b) If $h(x) = \arctan x \cdot g(x)$, what is $h'(1)$ and why?

5. Show why the derivative of $\ln x$ is what it is.

6. Show why the derivative of arcsin x is what it is.

7. Bunny is a calculus student at Enormous State University, and she's having some trouble. Bunny says "I think Calculus is totally unfair! It's like there's always a new thing so you never get to know it all, you know? So like there's this new function where its derivative is 1 over 1 plus x^2, right? But that was totally already the derivative of ln of 1 plus x^2, right? So it's like it's hopeless! There's no way you can understand 70% of something if they keep adding extra stuff!"

   Explain clearly to Bunny if there's anything she should understand better about the situation.

8. [Stewart] The half-life of radium-226 is 1590 years. A sample begins with a mass of 200 mg.
   a) Find a formula for the mass of radium-226 remaining after t years have elapsed.

   b) When (to the nearest year) will the sample be reduced to 150 mg of radium-226?

9. Evaluate $\lim_{x \to 0} \cot 2x \sin 6x$

10. Let $S(x) = \dfrac{e^x - e^{-x}}{2}$ and let $C(x) = \dfrac{e^x + e^{-x}}{2}$. What's interesting about the derivatives of $S(x)$ and $C(x)$?

Extra Credit (5 points possible):

Evaluate $\lim_{m\to\infty}\left(1+\dfrac{1}{m}\right)^m$ [Hint:You can almost use L'Hôpital's Rule on $\lim_{m\to\infty} m\ln\left(1+\dfrac{1}{m}\right)$, so try that for a start.]