Exam 4 Calc 1 11/12/2021

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. Find the critical numbers of $f(x) = x^2 - 6x + 2$.

2. Find the largest intervals on which $f(x) = x^3 - 6x^2 + 9x + 7$ is
   a) increasing
   b) decreasing

3. Find the largest intervals on which $f(x) = x^3 - 6x^2 + 9x + 7$ is
   a) concave up
   b) concave down

4. The sum of two numbers is 100. What is the greatest their product can be?

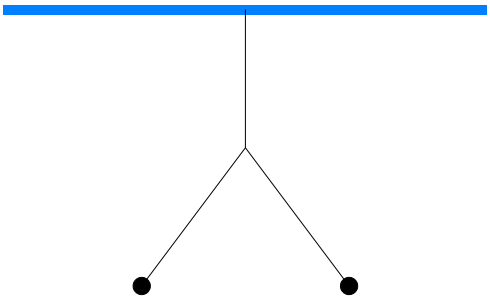
5. Find the absolute maximum and minimum values of $g(x) = 9 + 6x - x^2$ on [0,5].

6. If $f'(x) = 1 + 3\sqrt{x}$ and $f(4) = 25$, what is $f(x)$?

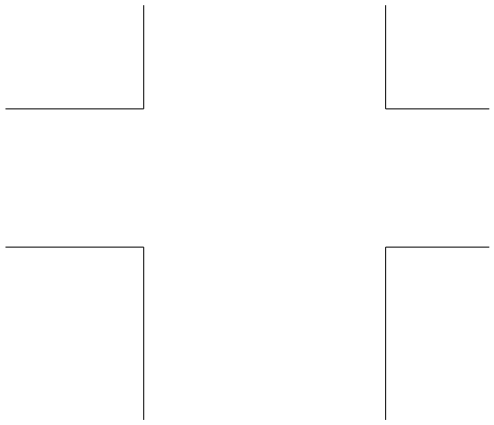
7. Biff is a calculus student at Enormous State University, and he's having some trouble. Biff says "Man, this calculus stuff is tough. I was doing pretty good with the critical stuff and maxes and stuff, but then this infliction point thing is just totally confusing. Is it just the same as the critical stuff or what?"

   Explain clearly to Biff what an inflection point is, how to find one, and why they might matter.

8. Two small towns are located 10 miles apart and directly east/west from each other. They intend to build a pumping station to draw water from an east/west flowing river 20 miles to the north, with a pipeline running south from the river to a junction point where the pipeline will branch, with one line running from the branch point to each town. How far south from the river should the branch point be located to minimize the total length of the pipeline?



9. The Boring Company has been digging 12-ft diameter tunnels, but has been considering upgrading to 21-ft diameter tunnels. If a 12-ft diameter tunnel meets a 21-ft diameter tunnel at right angles so that their central axes intersect, what is the longest length straight rod that could be fit around the corner from one tunnel to the other?



10. Find values for the constants a and b for which a function of the form $f(x) = a\,x^2\,e^{-b\,x}$ has a local maximum at (6,12).

Extra Credit (5 points possible): [Stewart] Find the point on the parabola $y = 1 - x^2$ at which the tangent line cuts from the first quadrant the triangle with smallest area.