

Exam 1 Calc 3 9/24/2021

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. State the formal definition of the partial derivative of a function $f(x, y)$ with respect to y .

2. Suppose that w is a function of x , y , and z , each of which is a function of t . Write the Chain Rule formula for $\frac{dw}{dt}$. Make very clear which derivatives are partials.

3. Write an equation for the plane tangent to $f(x, y) = x^2 - y/3 + 5$ at the point $(2, 6)$.

4. Show that $\lim_{(x,y) \rightarrow (0,0)} \frac{3xy}{x^2 + 2y^2}$ does not exist.

5. Let $f(x, y) = \sqrt{16 - x^2 - y^2}$. Find the maximum rate of change of f at the point $(1, 2)$ and the direction in which it occurs.

6. Show that for any vectors \vec{a} and \vec{b} , the vector $\vec{a} \times \vec{b}$ is perpendicular to \vec{b} .

7. Bunny is a calculus student at Enormous State University, and she's having some trouble. Bunny says "Ohmygod, this is the most totally confusing experience in my life. The professor told us there were these things we definitely had to know for the test, like in my notes I have that she said that the level curvy things are ninety degrees from the direction of greatest increase. And she said we have to know why that's true, but I totally don't have a clue. I looked in the book and it makes no sense at all. She never said anything about it in class, just during the review. So how am I supposed to know why it's true? This is so unfair!"

Explain clearly to Bunny how she could deduce such a conclusion from other things which she should indeed know.

8. [Stewart §11.6 #26] Suppose that you are climbing a hill whose shape is given by the equation $z = 1000 - 0.005x^2 - 0.01y^2$, where x , y , and z are given in meters, and you are standing at a point with coordinates $(60, 40, 966)$. The positive x -axis points east and the positive y -axis points north.

a. If you walk due south, will you start to ascend or descend? At what rate?

b. If you walk northwest, will you start to ascend or descend? At what rate?

9. Find the point(s) on the surface $4x^2 + 5y^2 + 5z^2 = 1$ at which the tangent plane is parallel to the plane $4x + 4y - 4z = 6$.

10. Find and classify (as maximum, minimum, or saddle point) all critical points of

$$f(x, y) = 2x^2 + 5y^2 + 2x^2y + 6$$

Extra Credit (5 points possible):

Let $f(x, y) = \sqrt{16 - x^2 - y^2}$. What's interesting about the direction in which the maximum rate of change of f occurs, and why?