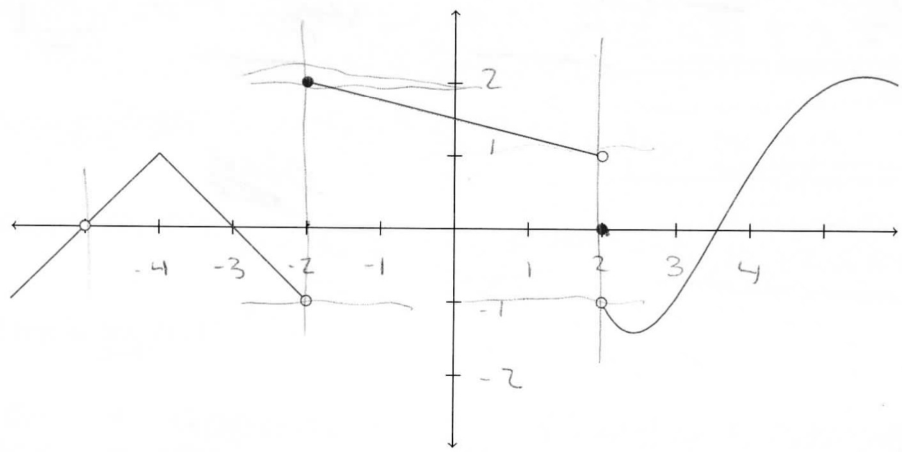
Exam 1b **Calc 1** **9/16/22**

Each problem is worth 10 points. For full credit provide good justification for your answers.

Use the graph of $f(x)$ for problems 1 and 2:



1. (a) What is $\lim_{x \to 2^+} f(x)$? -1

approaching ←

(b) What is $\lim_{x \to 2^-} f(x)$? 1

approaching →

(c) What is $\lim_{x \to 2} f(x)$? $-1 \neq 1$ so the limit DNE

(d) What is $\lim_{x \to -2^+} f(x)$? 2

approaching ←

(e) What is $\lim_{x \to -2^-} f(x)$? -1

approaching →

Great!

2. For which values of x does the function fail to be continuous?

f(x) fails to be continuous at x = -5 bc the output doesn't exist, x = -2 bc jump discontinuity, and x = 2 because neither 2^- or 2^+ side is approaching f(2).

Great!

3. Evaluate $\lim_{x \to 3} \frac{x^2 - 9}{x - 3}$.

$$\lim_{x \to 3} \frac{x^2-9}{x-3} = \lim_{x \to 3} \frac{\cancel{(x-3)}(x+3)}{\cancel{x-3}} = \lim_{x \to 3} x+3 = \boxed{6}$$

Good

4. Use the following table of values for $f(x)$ and $g(x)$ to find values for the following:

x	1	2	3	4	5	6
$f(x)$	6	3	5	1	4	2
$g(x)$	6	1	2	3	4	5

(a) $f(2)$ $= 3$

(b) $f(g(3))$ $= f(2)$
$= 3$

(c) $g(f(3))$ $= g(5)$
$= 4$

(d) $(f \circ g)(5)$ $= f(g(5))$
$= f(4) = 1$

(e) $(g \circ f)(5)$ $= g(f(5))$
$= g(4)$
$= 3$

Great!

5. Let $f(x) = \frac{\sin x}{x}$. **Make sure your calculator is in radian mode.** Give answers accurate to at least 8 decimal places.

(a) What is $f(0.2)$?

$f(.2) = \frac{\sin(.2)}{.2} = \underline{.993346654}$

(b) What is $f(0.1)$?

$f(.1) = \frac{\sin(.1)}{.1} = \underline{.9983341665}$

(c) What is $f(0.01)$?

$f(.01) = \frac{\sin(.01)}{.01} = \underline{.9999833334}$

(d) What is $f(0.001)$?

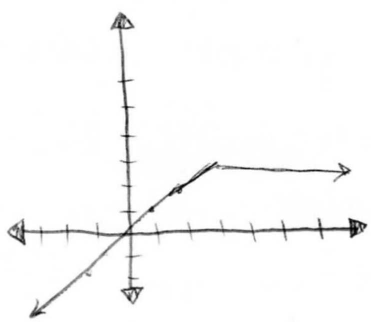
$f(.001) = \frac{\sin(.001)}{.001} = \underline{.999999833}$

(e) What is $\lim_{x \to 0} f(x)$?

although, f(0) is undefined because $\frac{0}{0}$ = undefined.
the limits from both left and right agree
the limit is 1

Excellent!

6. For what value(s) of c is $h(x) = \begin{cases} x & \text{if } x < 3 \\ c & \text{if } x \geq 3 \end{cases}$ continuous?



f(x)	x
	[illegible]
2	2
1	1

to be continuous
c must equal $\boxed{3}$

Great!

7. Biff is a calculus student at Enormous State University, and he's having some trouble. Biff says "Crap. Calc is totally killing me. I thought I knew it all from high school, but now there's all this limit crap and you can't just put it in your calculator once, they say you gotta try a bunch of things. So like, I tried 3.9 and 3.99 for approaching 4 on this quiz, and it was getting super close to 0, like it was 0.03 then 0.003, right? So I said the limit was 0, but they said I needed to try more. How the eff many more?"

Help Biff by explaining as clearly as you can what else he really should have checked, and why.

He should have checked the other side of 4 as well. For example 4.01, 4.001 just to make sure is calculations is correct and that there is a limit. He can also double check his answer by plotting in a graph.

Excellent plan!

8. Over the first few seconds after a chunk of frozen urine falls off the vent of a Russian surveilance plane flying over Ukraine, the height (in feet) of the chunk is given by the function $h(t) = 30,000 - 16t^2$. What is the chunk's average velocity over the time period beginning when $t = 3$ and lasting

(a) 0.5 seconds

$h(3) = 30,000 - 16(3)^2 = 29,856$

$h(3.5) = 30,000 - 16(3.5)^2 = 29,804$

$$\frac{29,804 - 29,856 \text{ ft}}{3.5 - 3 \text{ s}} = \boxed{-104 \text{ ft/s}}$$

(b) 0.1 seconds

$h(3.1) = 30,000 - 16(3.1)^2 = 29,846.24$

$$\frac{29,846.24 - 29,856}{3.1 - 3} = \boxed{-97.6 \text{ ft/s}}$$

(c) 0.01 seconds

$h(3.01) = 30,000 - 16(3.01)^2 = 29855.0384$

$$\frac{29,855.04 - 29,856}{3.01 - 3} = \boxed{-96 \text{ ft/s}}$$ (or -96.16)

Excellent

9. Given the function $f(x) = \frac{1}{2x}$, simplify $\frac{f(a+h)-f(a)}{h}$.

$f(a) = \frac{1}{2a}$

$f(a+h) = \frac{1}{2(a+h)}$

Now,

$\frac{f(a+h)-f(a)}{h}$

$= \frac{\frac{1}{2(a+h)} - \frac{1}{2a}}{h}$

$= \frac{2a-(2a+2h)}{4a(a+h)} \times \frac{1}{h}$

$= \frac{\cancel{2a}-\cancel{2a}-2h}{4ah(a+h)}$

$= \frac{-2\cancel{h}}{4a\cancel{h}(a+h)}$

$= -\frac{1}{2a(a+h)}$

Excellent!

10. Evaluate $\lim_{x\to\infty}\left(\sqrt{x^2+ax}-\sqrt{x^2+bx}\right)$. $= \lim_{x\to\infty} \dfrac{\sqrt{x^2+ax}-\sqrt{x^2+bx}}{1}\cdot\dfrac{\sqrt{x^2+ax}+\sqrt{x^2+bx}}{\sqrt{x^2+ax}+\sqrt{x^2+bx}}$

$$= \lim_{x\to\infty} \frac{(x^2+ax)-(x^2+bx)}{\sqrt{x^2+ax}+\sqrt{x^2+bx}}$$

$$= \lim_{x\to\infty} \frac{ax-bx}{\sqrt{x^2+ax}+\sqrt{x^2+bx}} \cdot \frac{\frac{1}{x}}{\frac{1}{x}}$$

$$= \lim_{x\to\infty} \frac{a-b}{\sqrt{1+\frac{a}{x}}+\sqrt{1+\frac{b}{x}}}$$

$$= \frac{a-b}{1+1}$$

$$= \frac{a-b}{2}$$