## Exam 2a Calc $1 \quad$ 10/7/22

Each problem is worth 10 points. For full credit provide good justification for your answers.

1. State the formal definition of the derivative of a function $f(x)$.
2. Use the definition of the derivative to find the derivative of $f(x)=\sqrt{x}$.
3. Use the following table of values for $f(x)$ and $g(x)$ to find values for the following:

| $x$ | 1 | 2 | 3 | 4 | 5 | 6 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| $f(x)$ | 6 | 3 | 5 | 1 | 4 | 2 |
| $g(x)$ | 2 | 1 | 6 | 3 | 5 | 4 |
| $f^{\prime}(x)$ | 2 | 4 | 1 | 5 | 7 | 8 |
| $g^{\prime}(x)$ | 5 | 9 | 7 | 11 | 2 | 12 |

(a) If $h(x)=f(x) \cdot g(x)$, what is $h^{\prime}(1)$ and why?
(b) If $h(x)=\frac{f(x)}{g(x)}$, what is $h^{\prime}(6)$ and why?
(c) If $h(x)=f(g(x))$, what is $h^{\prime}(4)$ and why?
(d) If $h(x)=5 \cdot g(x)+20$, what is $h^{\prime}(3)$ and why?
(e) If $h(x)=f\left(x^{2}\right)$, what is $h^{\prime}(2)$ and why?
4. Prove that $(f+g)^{\prime}(x)=f^{\prime}(x)+g^{\prime}(x)$ for any differentiable functions $f$ and $g$.
5. [Stewart] A cylindrical tank with radius 5 m is being filled with water at a rate of 3 $\mathrm{m}^{3} / \mathrm{min}$. How fast is the height of the water increasing?
6. State and prove the Product Rule.
7. Bunny is a calculus student at Enormous State University, and she's having some trouble. Bunny says "OMG! Why does calculus have to be so confusing, like, they're literally trying to kill us? The professor definitely said that the derivative of 5 is zero on Monday, but then totally said that the derivative of 5 is 5 on Wednesday. Is it really different on different days?"

Help Bunny by explaining as clearly as you can what's going on.
8. Find the local linearization for $f(x)=\sqrt{x}$ at $a=4$.
9. Find an equation for the line tangent to $f(x)=\sqrt[3]{x}$ at the point $(0,0)$.
10. Find an equation of the line tangent to $x^{2}+3 x y-y^{2}=3$ at the point $(1,2)$.


Extra Credit (5 points possible):
The graph of the equation in $\# 10$ has two different pieces to it. Do the tangent lines to the piece on the right ever hit the piece on the left, or vice versa? How can you tell?

