**Exam 3b** **Calc 1** **10/26/22**

Each problem is worth 10 points. For full credit provide good justification for your answers.

1. Suppose that $f(x) = (15 - \ln(x))^8$. Find $f'(2)$.

$$f(x) = (15 - \ln(x))^8$$

$$f'(x) = 8(15-\ln(x))^7 \cdot (0 - \tfrac{1}{x})$$

$$= 8(15-\ln(x))^7 \times -\frac{1}{x}$$

$$= -\frac{8(15-\ln(x))^7}{x}$$

Great

$$f'(2) = -\frac{{}^4 8(15 - \ln 2)^7}{2}$$

$$= -4(15-\ln 2)^7 \;_{\prime\prime}$$

2. Let $g(x) = \arctan(x^2)$. What is $g'(x)$?

$$\rightarrow \quad g(x) = \arctan(x^2)$$

$$g'(x) = \frac{1}{1+(x^2)^2} \times 2x \qquad \left[ \text{Using } \tan^{-1}(x) = \frac{1}{1+x^2} \right.$$

and chain Rule ].

$$= \frac{2x}{1+x^4} \quad \#$$

Excellent!

3. Use the following table of values for $f(x)$ and $g(x)$ to find values for the following:

| $x$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $f(x)$ | 6 | 3 | 5 | 1 | 4 | 2 |
| $g(x)$ | 2 | 1 | 6 | 3 | 5 | 4 |
| $f'(x)$ | 2 | 4 | 1 | 5 | 7 | 8 |
| $g'(x)$ | 5 | 9 | 7 | 11 | 2 | 12 |

(a) If $h(x) = e^{(f(x))}$, what is $h'(2)$ and why?

$h'(x) = e^{(f(x))} \cdot f'(x)$ Chain Rule!

$h'(2) = e^{f(2)} \cdot f'(2) = e^3 \cdot 4$

(b) If $h(x) = \ln(g(x))$, what is $h'(3)$ and why?

Chain Rule!

$h'(x) = \frac{1}{g(x)} \cdot g'(x)$

$h'(3) = \frac{1}{g(3)} \cdot g'(3) = \frac{1}{6} \cdot 7$

(c) If $h(x) = g(x) \cdot \tan^{-1} x$, what is $h'(4)$ and why?

Product Rule!

$h'(x) = g'(x) \cdot \tan^{-1} x + g(x) \cdot \frac{1}{1+x^2}$

$h'(4) = g'(4) \cdot \tan^{-1} 4 + g(4) \cdot \frac{1}{1+(4)^2} = 11 \cdot \tan^{-1} 4 + 3 \cdot \frac{1}{17}$

4. Why is the derivative of $\ln x$ equal to $\frac{1}{x}$?

$(\ln x)' = 1/x$

$y = \ln x$

$e^y = e^{\ln x}$

$e^y = x$

Differentiate

$e^y \cdot y' = 1$

$y' = \frac{1}{e^y}$

$y' = \frac{1}{e^{\ln x}}$

$y' = \frac{1}{x}$

therefore

$(\ln x)' = \frac{1}{x}$

✓

Nice!

y is equal to ln x as shown above, therefore y' is equal to (ln x)'

5. Find $\lim\limits_{x \to 1} \dfrac{\ln x}{x-1}$.

$$= \lim_{x \to 1} \frac{\ln x}{x-1}$$ cannot dividing by zero

$$\overset{LH}{=} \lim_{x \to 1} \frac{\frac{1}{x}}{1}$$ Good!

$$= \lim_{x \to 1} \frac{1}{x}$$ table confirms

$$= \frac{1}{1}$$

$$= 1$$

| x | y |
|---|---|
| .98 | 1.01 |
| 1 | error |
| 1.02 | .99 |

6. Why is the derivative of $\sin^{-1} x$ equal to $\frac{1}{\sqrt{1-x^2}}$?

① I Know : $\sin(\arcsin x) = x$

② Differentiate : $\cos(\arcsin x) \cdot (\arcsin x)' = 1$

$$(\arcsin x)' = \frac{1}{\cos(\arcsin x)}$$

③


$\cos(\arcsin x) = \frac{\sqrt{1-x^2}}{1}$

$$\boxed{(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}}$$

Good!

7. Bunny is a calculus student at Enormous State University, and she's having some trouble. Bunny says "OMG! Why does calculus have to be so confusing, like, they're literally trying to kill us? We pay like $380 for the stupid book and online homework and all it does is tell us we're wrong. I'm sure I did it right, because when you plug the number in you get a number, right? And then when you do the derivative of just a number it's always zero, right? So every question they asked I got 0 for, but it counted them all wrong. Is it just that they can't admit the answers are all zero because then they wouldn't have a job and be able to make us give them all that money?"

Help Bunny by explaining as clearly as you can why her approach might not be valid in all cases.

Bunny should do the derivative of the function first and then plug in the number. Because she plugged the number in first, of course the derivative is 0 as the derivative of constant is always 0. Therefore she should find the derivative before plugging in the number.

Excellent

8. [Stewart] A sample of tritium-3 decayed to 94.5% of its original amount after a year.

(a) What is the half-life of tritium-3?

(b) How long would it take the sample to decay to 30% of its original amount?

$(1, .945A)$ $(0, A)$

$F(x) = A \cdot b^x$ $\qquad \frac{.945A}{A} = \frac{A}{A} \cdot b^1$

$A = A \cdot b^0$ $\qquad .945 = b$

$A = A$

a) $f(x) = A \cdot .945^x$

$\frac{1}{2}A = A \cdot 0.945^x$

b) $(t, .30A)$ $\qquad \frac{1}{2} = 0.945^x$

$\log_{0.945} \frac{1}{2} = \log_{0.945} x$

$.30A = A \cdot .945^x$ $\qquad \log_{0.945} \frac{1}{2} = x$

$.30 = 0.945^x$ (log) $\qquad$ a) $\boxed{x = 12.2528 \text{ yrs}}$

$\log_{0.945} .30 = x$

b) $\boxed{x = 21.2827 \text{ yrs}}$

Good

9. [Anton] Suppose that the population of oxygen-dependant bacteria in a pond is modeled by the equation

$$P(t) = \frac{60}{5 + 7e^{-t}}$$

where $P(t)$ is the population (in billions) $t$ days after an initial observation at time $t = 0$. At what rate is the population changing 3 days after the initial observation?

$$P'(t) = \frac{0 \cdot (5 + 7e^{-t}) - 60(-7e^{-t})}{(5 + 7e^{-t})^2}$$

So $P'(3) = \dfrac{420\, e^{-3}}{(5 + 7e^{-3})^2} \approx 0.73$ billion/day growth!

10. (a) What is the slope of the line tangent to $f(x) = \tan^{-1}(x - x^2)$ at the point where $x = 2$?

(b) Where is the line tangent to $f(x)$ horizontal?

$$f'(x) = \frac{1}{1+(x-x^2)^2} \cdot (1-2x)$$

$$\text{So} \quad f'(2) = \frac{1}{1+(2-2^2)^2} \cdot (1-2\cdot 2) = \frac{1}{5} \cdot -3 = \frac{-3}{5} \text{ is } m$$

Horizontal is where slope is 0, so

$$0 = \frac{1-2x}{\text{stuff}}$$

$$\text{stuff} \cdot 0 = 1-2x$$

$$0 = 1-2x$$

$$2x = 1$$

$$x = 1/2$$

The tangent line is horizontal where $x = 1/2$