Exam 4a **Calc 1** **11/11/22**

Each problem is worth 10 points. For full credit provide good justification for your answers.

1. Find the critical numbers of $f(x) = x^3 - 9x^2 + 24x$.

$f'(x) = 3x^2 - 18x + 24$

$0 = 3(x^2 - 6x + 8)$

$0 = 3(x-4)(x-2)$

$x = 4 \quad x = 2$

$(x-4)(x-2)$

$x^2 - 2x - 4x + 8$

Good

The critical numbers of $f(x) = x^3 - 9x^2 + 24x$ are when $x=4$ and $x=2$.

2. Find the interval(s) on which $f(x) = 6x^2 - 36x$ is increasing.

$f'(x) = 12x - 36$

$0 = 12x - 36$

$12x = 36$

$x = 3$

interval	$f'(x)$
$(-\infty, 3)$	$-$
$(3, \infty)$	$+$

Excellent

$f'(0) = 12(0) - 36$
$= -36$

$f'(4) = 12(4) - 36$
$= 48 - 36$
$= 12$

$f(x) = 6x^2 - 36x$ is increasing on the interval $(3, \infty)$

3. Find the most general antiderivatives of the the following functions:

(a) $f(x) = x^n$

$$F(x) = \frac{1}{n+1} x^{n+1} + C$$

(b) $f(x) = e^x$

$$F(x) = e^x + C$$

(c) $f(x) = \sin x$

$$F(x) = -\cos x + C$$

(d) $f(x) = \sec^2 x$

$$F(x) = \tan x + C$$

(e) $f(x) = \frac{1}{x}$

$$F(x) = \ln|x| + C$$

4. Let $f(x) = x^2 - 6x + 3$. Find the absolute maximum and absolute minimum values of f on $[0, 5]$.

$f'(x) = 2x - 6$

$0 = 2x - 6$

$\frac{6}{2} = \frac{2x}{2}$ $x = 3.$

$f(0) = 0^2 - 6(0) + 3 = 3$

$f(3) = 3^2 - 6(3) + 3 = -6$

$f(5) = 5^2 - 6(5) + 3 = -2$

0 3 5

$\boxed{\begin{array}{l}\text{Max. value} = 3 \\ \text{Min value} = -6\end{array}}$

Great

5. Let $f(x) = 6x - 2x^3$. Find all intervals where f is concave down.

second derivative

1) Differentiate:

$6 - 6x^2$

2) Differentiate again

$-12x$

Concave down: $(0, \infty)$

3) Solve for zero:

$0 = -12x$

$0 = x$

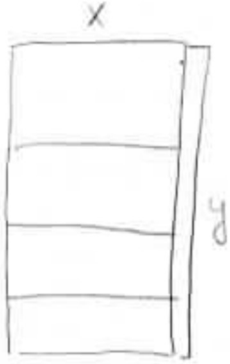
Excellent

4) Test above/below critical:

$-12(1) = -12$

$-12(-1) = 12$

6. A komodo dragon farmer has 2400 ft of fencing and wants to fence off a rectangular field and then divide it into 4 equal parts with fencing parallel to one side of the rectangle. What are the dimensions of the field that has the largest area?



$$2400 = 5x + 2y$$

$$2y = 2400 - 5x$$

$$y = 1200 - \frac{5}{2}x$$

$$A(x) = x \cdot y$$

$$A(x) = x\left(1200 - \frac{5}{2}x\right)$$

$$A(x) = 1200x - \frac{5}{2}x^2$$

$$A'(x) = 1200 - 5x$$

$$0 = 1200 - 5x$$

$$5x = 1200$$

$$x = 240$$

Excellent!

$$y = 1200 - \frac{5}{2}(240)$$

$$y = 600$$

240 ft × 600 ft

7. [Stewart] A baseball team plays in a stadium that holds 55,000 spectators. With ticket prices at \$10, the average attendance had been 27,000. When ticket prices were lowered to \$8, the average attendance rose to 33,000. How should ticket prices be set to maximize revenue?

$10 - 8 = 2$ dollar

$33000 - 27000 = 6000$

So, when lower 1 dollar, there are more 3000 attendance! Yes!

$f(x) = (10 - x)(27000 + 3000x)$

$= 270000 + 30000x - 27000x - 3000x^2$

$= -3000x^2 + 3000x + 270000$

$f'(x) = -6000x + 3000$

Let $f'(x) = 0$

$-6000x + 3000 = 0$

$x = \frac{1}{2}$

Great

$10 - \frac{1}{2} = 9.5$

So ticket price is 9.5

8. The owner of the baseball team from Problem #8 is named Biff, and he says "What the [bleep] are we doing with this attendance [bleep]? Why in the holy [bleep] did I pay for a 55,000 seat stadium if we're only gonna have half that many people there? Do you think I'm a [bleep] idiot? Figure out what price to sell the [bleep] tickets for to fill every [bleep] seat in my [bleep] stadium so I can make my money back!

Help Biff by explaining as clearly as you can whether filling all the seats in his stadium is as good an idea as he thinks, and why.

Dear Biff, Filling all of the seats in the stadium won't give us maximum revenue. At some point in order to get the seats full, we would have to drop the ticket prices beyond a lucrative margin. Finding the maximum conditions of revenue allows us to make the most amount of money possible because it balances the cost of a ticket so that it's affordable enough for lots of people.

Good

9. Let $y = \dfrac{\sin x}{2+\cos x}$. Find the exact coordinates of the lowest point on this graph in the interval $[0, 2\pi]$.

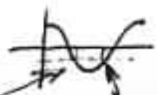
$$y' = \frac{\cos x\,(2+\cos x) - \sin x \cdot -\sin x}{(2+\cos x)^2}$$

$$y' = \frac{2\cos x + \cos^2 x + \sin^2 x}{(2+\cos x)^2}$$

$$0 = \frac{2\cos x + 1}{(2+\cos x)^2}$$

$$0 = 2\cos x + 1$$

$$\cos x = \frac{-1}{2}$$



$$x = \frac{2\pi}{3} \text{ and } \frac{4\pi}{3}$$

$$y = \frac{\sin\left(\frac{4\pi}{3}\right)}{2+\cos\left(\frac{4\pi}{3}\right)} = \frac{-\frac{\sqrt{3}}{2}}{2+\frac{-1}{2}} = \frac{\sqrt{3}}{2}\cdot\frac{2}{3} = \frac{\sqrt{3}}{3}$$

So the low one is $\left(\frac{4\pi}{3}, \frac{\sqrt{3}}{3}\right)$

so it's close to

$\approx (4.1888, 0.5774)$

10. [Anton] Suppose that the population of oxygen-dependant bacteria in a pond is modeled by the equation

$$P(t) = \frac{60}{5 + 7e^{-t}}$$

where $P(t)$ is the population (in billions) t days after an initial observation at time $t = 0$. What can you say about when the population is growing at the greatest rate?

$$P'(t) = \frac{0 \cdot (5 + 7e^{-t}) - 60 \cdot -7e^{-t}}{(5+7e^{-t})^2}$$

$$P'(t) = \frac{420e^{-t}}{(5+7e^{-t})^2}$$

That's the rate it's growing, so to maximize we take the derivative of that and set it to 0:

$$P''(t) = \frac{-420e^{-t}(5+7e^{-t})^2 - 420e^{-t}(2(5+7e^{-t}) \cdot -7e^{-t}}{(5+7e^{-t})^4}$$

$$0 = -420e^{-t}(5+7e^{-t})[(5+7e^{-t}) - 14e^{-t}]$$

$$0 = \underbrace{-420e^{-t}}_{\text{never } 0}\underbrace{(5+7e^{-t})}_{\text{never } 0}\underbrace{(5-7e^{-t})}$$

$$5 - 7e^{-t} = 0$$

$$5 = 7e^{-t}$$

$$5/7 = e^{-t}$$

$$\ln 5/7 = -t$$

$$t = -\ln(5/7)$$

so about a third of a day later