Exam 4b **Calc 1** **11/11/22**

Each problem is worth 10 points. For full credit provide good justification for your answers.

1. Find the critical numbers of $f(x) = x^3 - 12x^2 + 36x$.

2. Find the x coordinates of any inflection point(s) of $f(x) = x^3 - 12x^2 + 36x$.

3. Find the most general antiderivatives of the the following functions:

(a) $f(x) = x^n$

(b) $f(x) = \sin x$

(c) $f(x) = e^x$

(d) $f(x) = \dfrac{1}{\sqrt{1-x^2}}$

(e) $f(x) = \dfrac{1}{x}$

4. Let $f(x) = x^2 - 4x + 3$. Find the absolute maximum and absolute minimum values of f on $[0, 5]$.

5. Let $f(x) = 6x - 2x^3$. Find the largest possible interval(s) where f is decreasing.

6. [Stewart] A farmer has 2400 ft of fencing and wants to fence off a rectangular field that borders a straight river. He needs no fence along the river. What are the dimensions of the field that has the largest area?

7. Bunny is a calculus student at Enormous State University, and she's having some trouble. Bunny says "OMG! Why do they make it so confusing? I get the slopey parts, you know? But then they have this cavity part, which makes no sense because that's teeth, right? But so somehow the cavity tells you a max instead of a min or something, right? What's up with that?"

   Help Bunny by explaining as clearly as you can how concavity connects to maxes and mins.

8. Let $y = \frac{1}{2}x - \sin x$. Find the exact coordinates of the lowest point on this graph in the interval $[0, 2\pi]$.

9. Rectangular storage bins are to be made with square bases and open tops. The volume of each bin is to be 1 cubic meter. What dimensions use the least (in terms of square meters) amount of material?

10. [Anton] Suppose that the population of oxygen-dependant bacteria in a pond is modeled by the equation

$$P(t) = \frac{60}{5 + 7e^{-t}}$$

where $P(t)$ is the population (in billions) t days after an initial observation at time $t = 0$. What can you say about when the population is at a maximum?

Extra Credit (5 points possible): Jon can run 10 meters per second on the straight road running north through the jungle, but then will veer off into the thick underbrush to reach a secret trapdoor which leads to an ancient temple filled with priceless acrheological treasures. In the thick underbrush Jon can only run 5 meters per second. The trapdoor is at a point precisely 100 feet west of a spot on the road 300 feet north from Jon's current position. Determine exactly where Jon should veer off the road to reach the trapdoor (and immortal fame) as quickly as possible, because there are angry tribal warriors chasing him.