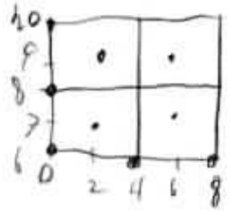
Exam 2 **Calc 3** **10/28/22**

Each problem is worth 10 points. For full credit provide good justification for your answers.

1. Write a double Riemann sum for $\iint_R f\,dA$, where $R = \{(x,y) : 0 \le x \le 8, 6 \le y \le 10\}$ using midpoints with $n = m = 2$ subdivisions



$\Delta x = 4$
$\Delta y = 2$

$\Delta x \cdot \Delta y \, (\quad)$

$8(f(2,7) + f(6,7) + f(2,9) + f(6,9))$

$\Delta x \cdot \Delta y = 4 \cdot 2 = 8$

Good

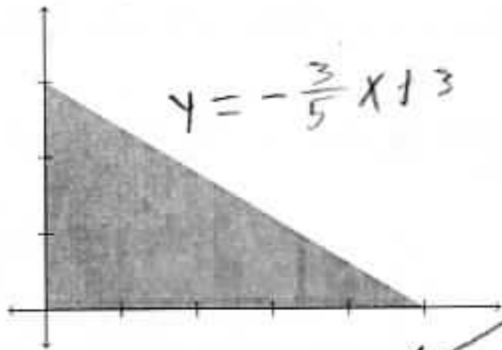
$(2,7), (6,7), (2,9)(6,9)$

2. Set up a double integral for the integral from #1.

$$\int_{y=6}^{y=10} \int_{x=0}^{x=8} (f)\,dx\,dy$$

Good.

3. Set up limits of integration for finding the volume under $f(x, y) = 5 + x + 2y$ within the pumpkin-colored region shown:

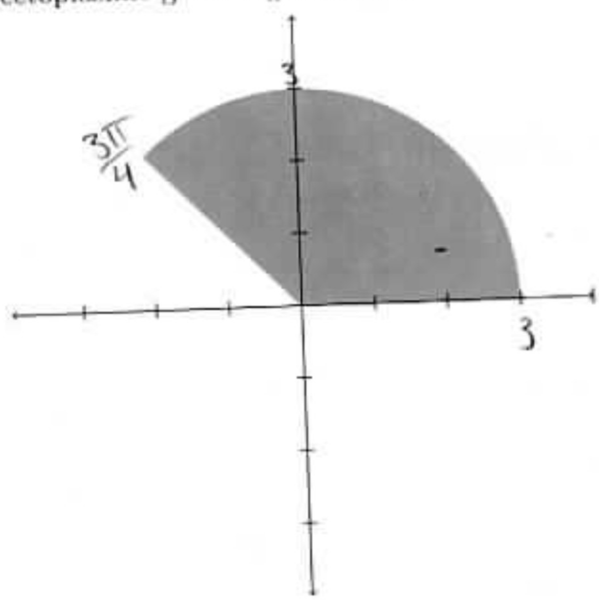


$$\int_0^5 \int_0^{-\frac{3}{5}x+3} \int_0^{5+x+2y} 1 \, dz \, dy \, dx$$

$$\int_0^5 \int_0^{-\frac{3}{5}x+3} 5 + x + 2y \, dy \, dx$$

Great

4. Set up limits of integration for finding the volume under $g(x, y) = 10 - x$ within the ectoplasmic green region shown:



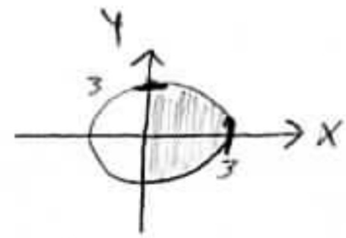
$$\int_0^{\frac{3\pi}{4}} \int_0^3 (10 - x) \, r \, dr \, d\theta$$

no x's

$$\int_0^{\frac{3\pi}{4}} \int_0^3 (10 - r\cos(\theta)) \, r \, dr \, d\theta$$

Good

5. Evaluate $\int_0^3 \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \int_0^{\sqrt{9-x^2-y^2}} 2\, dz\,dy\,dx$



$x^2 + y^2 + z^2 = 9$

This is one fourth the volume of a sphere with radius 3 multiplied by 2.

Excellent

$2\left(\frac{1}{4}\right)\frac{4}{3}\pi(3)^3 = 18\pi$

6. Show that the Jacobian for the conversion from rectangular to polar coordinates is what it is.

$x = r\cos(\theta)$

$y = r\sin(\theta)$

$$J = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial y}{\partial r} \\ \frac{\partial x}{\partial \theta} & \frac{\partial y}{\partial \theta} \end{vmatrix} = \begin{vmatrix} \cos(\theta) & \sin(\theta) \\ -r\sin(\theta) & r\cos(\theta) \end{vmatrix}$$

$= r\cos^2(\theta) - -r\sin^2(\theta)$

Great

$= r\cos^2(\theta) + r\sin^2(\theta)$

$= r\underbrace{\left(\cos^2(\theta) + \sin^2(\theta)\right)}_{=1 \text{ by Pyth}}$

$\boxed{= r}$

7. Bunny is a calculus student at Enormous State University, and she's having some trouble. Bunny says "OMG! These spherical thingies are so hard! I tried to do, like, the online homework, right? And I put limits of 0 to 2π on everything with angles because this one guy who was working in the computer lab told me that's pretty much always right? And the theta ones it was right on a lot of them, but on the phi ones it wasn't right on *any* of them! Can you believe how unfair that is?"

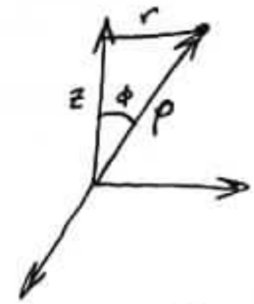
Help Bunny by explaining as clearly as you can why the responses she got make sense.

Bunny, your theta limits were usually right because it is common to integrate an entire sphere. Spherical coordinates are defined such that letting θ range from 0 to 2π and ϕ range from 0 to π allows you to cover all of a sphere. ϕ, which is a vertical angle measured from the positive z-axis, would be covering the sphere twice if allowed to range from 0 to 2π. For example, this is why latitude only spans a range of 180° or π radians.

Excellent

8. Set up an integral for the volume of the solid that lies above the cone $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = 4$.

$z = \sqrt{r^2} = r$



$$\int_0^{2\pi} \int_0^{\frac{\pi}{4}} \int_0^{2} 1\, \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

$\sin\phi = \frac{r}{\rho}$

so $r = \rho \sin\phi$

$\cos\phi = \frac{z}{\rho}$

so $z = \rho\cos\phi$

Then

$z = r$

becomes

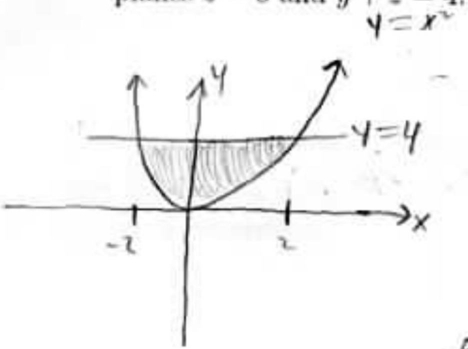
$\rho\cos\phi = \rho\sin\phi$

$1 = \frac{\sin\phi}{\cos\phi}$

$1 = \tan\phi$

$\phi = \frac{\pi}{4}$

9. Set up an integral for the volume of the solid enclosed by the cylinder $y = x^2$ and the planes $z = 0$ and $y + z = 4$.

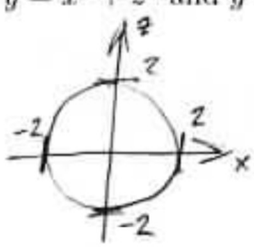


$$Z = 4 - Y$$

$$V = \int_{-2}^{2}\int_{x^2}^{4}\int_{0}^{4-y} 1 \; dZ\, dY\, dx$$

Excellent

10. Set up an integral to integrate $f(x, y, z) = y$ over the solid enclosed by the paraboloids $y = x^2 + z^2$ and $y = 8 - x^2 - z^2$.



Intersection: $x^2 + z^2 = 8 - x^2 - z^2$

$$2x^2 + 2z^2 = 8$$

$$x^2 + z^2 = 4$$

$$\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{x^2+z^2}^{8-x^2-z^2} y \, dy\, dz\, dx$$