Exam 3 **Calc 3** **11/29/22**

Each problem is worth 10 points. For full credit provide good justification for your answers.

1. Give a parametrization and bounds for t to produce a counterclockwise circle with radius 75 centered at the origin.

2. Is the vector field $\mathbf{F}(x, y) = \langle xy^2, x + y^2 \rangle$ conservative? How can you be sure?

3. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F}(x, y) = 5\mathbf{i} + 3x\mathbf{j}$ and C is a path composed of a line segment from $(0, 0)$ to $(2, 0)$ followed by a line segment from $(2, 0)$ to $(0, 1)$ and then a line segment from $(0, 1)$ back to $(0, 0)$.

4. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F}(x, y) = 2xy^3\mathbf{i} + 3x^2y^2\mathbf{j}$ and C is a line segment from $(1, 1)$ to $(2, 3)$.

5. Let $\mathbf{F}$ be the vector field $\mathbf{F}(x, y, z) = 2x\mathbf{i} + 5y\mathbf{j} + 3z\mathbf{k}$. Let S be the sphere with radius 3, centered at the origin and oriented outward. Find $\iint_S \mathbf{F} \cdot d\mathbf{S}$.

6. Show that for any vector field $\mathbf{F}(x, y, z)$ whose component functions have continuous partial derivatives, $\text{div}(\text{curl } \mathbf{F}) = 0$. Make it clear how the requirement that the partials be continuous is important.

7. Biff is a calculus student at Enormous State University, and he's having some trouble. Biff says "Crap. This line integral stuff is crazy. There's all these different ways, and at first I thought I'd just do them all the first way they showed us, so I kinda ignored a couple of days, but then it get pretty hard. They were saying something yesterday about a super-shortcut, where like, if the vector field was the right kind then for any closed path the answer is automatically zero. I like that, but what is it about the vector field that tells you when that happens?"

   Help Biff by explaining as clearly as you can when you can tell that a line integral on any closed path has to come out to zero, and how you know.

8. Let $\mathbf{F}$ be the vector field $\mathbf{F}(x, y, z) = \langle x, y, 1\rangle$. Let S be the cylinder $x^2 + y^2 = 4$ between $z = 0$ and $z = 5$. Find $\displaystyle\iint_S \mathbf{F} \cdot d\mathbf{S}$.

9. Let $\mathbf{F}$ be the vector field $\mathbf{F}(x, y, z) = \langle xz, y^2, -x^3 \rangle$. Let S be the portion of the paraboloid $z = x^2 + y^2$ below $z = 4$ with upward orientation. Find $\iint_S \operatorname{curl} \mathbf{F} \cdot d\mathbf{S}$.

10. Let $\mathbf{F}(x, y) = \langle 1, 0 \rangle$, and let C be the top half of a circle with some radius r centered at the origin, traversed counterclockwise from $(r, 0)$ to $(-r, 0)$. For what radius r will $\displaystyle\int_C \mathbf{F} \cdot d\mathbf{r} = 5$?

Extra Credit (5 points possible): Show that if $f(x, y, z)$ is a scalar function and $\mathbf{G}(x, y, z)$ is a vector field, then

$$\operatorname{div}(f\mathbf{G}) = f \operatorname{div} \mathbf{G} + \mathbf{G} \cdot \nabla f$$