Exam 1 **Real Analysis 1** **9/28/22**

Each problem is worth 10 points. Show adequate justification for full credit. Don't panic.

1. State the definition of a function $f(x)$ converging to a limit L as x approaches a.

2. (a) State the definition of s_0 being an accumulation point of a set S.

   (b) Give an example of a subset of $\mathbb{R}$ with infinitely many elements, but no accumulation points.

3. Give an example of a sequence which converges, but is not monotone.

4. Show that the limit of a sequence, if it exists, is unique.

5. If $\{a_n\}$ is a Cauchy sequence and $S = \{a_n | n \in \mathbb{N}\}$ is finite, then $\{a_n\}$ is constant from some point on.

6. State and prove the Bolzano-Weierstrass Theorem for Sets.

7. Show that if $\lim_{x \to \infty} f(x) = A$ and $\lim_{x \to \infty} g(x) = B$, then $\lim_{x \to \infty} (f \cdot g)(x) = AB$

8. State and prove the Monotone Convergence Theorem.

9. We say that a sequence $\{a_n\}$ is **convergish** to L iff there exists an $\epsilon > 0$ for which there exists an n^* such that $n > n^*$ implies $|a_n - L| < \epsilon$. When a sequence is convergish, we call the L involved a **limish**. Prove or give a counterexample: If a sequence is convergish, then its limish is unique.

10. (a) Show that $\lim_{x \to a} x = a$.

(b) Show that for any $n \in \mathbb{N}$, $\lim_{x \to a} x^n = a^n$