Exam 2 **Real Analysis 1** **11/4/22**

Each problem is worth 10 points. Show adequate justification for full credit. Don't panic.

1. State the definition of the derivative of a function $f(x)$ at $x = a$.

2. a) State the definition of a function f being continuous at $x = a$.

b) State the definition of a function f being continuous on D.

3. State the Mean Value Theorem

4. a) State the definition of a compact set.

b) State the Heine-Borel Theorem.

c) Give an example of an open cover for $(0, 2022)$ that has no finite subcover.

5. State and prove the Product Rule for Derivatives, making clear how your hypotheses are necessary.

6. Prove that the sum of continuous functions is continuous.

7. State and prove the Extreme Value Theorem.

8. State and prove Rolle's Theorem.

9. a) Prove or give a counterexample: If f is bounded, then f' is bounded.

b) Prove or give a counterexample: If f' is bounded, then f is bounded.

10. Let E be a set, with $E \subseteq \mathbb{R}$. Show that if E is open, then $\mathbb{R} \backslash E$ is closed.