Exam 3 Calc 1 10/27/23

Each problem is worth 10 points. For full credit provide good justification for your answers.

1. Find the derivative of $f(x) = \log_5 x + 3^x$.

2. Find the derivative of $g(x) = \ln(x^3 + 1)$.

3. Use the following table of values for f(x) and g(x) to find values for the following:

x	1	2	3	4	5	6
f(x)	5	4	6	1	3	2
g(x)	1	6	2	3	5	4
f'(x)	2	3	4	5	6	1
g'(x)	2	7	3	13	11	8

(a) If $h(x) = \tan^{-1}(f(x))$, what is h'(3) and why?

(b) If $h(x) = g(x) \cdot \ln x$, what is h'(5) and why?

(c) If $h(x) = \arcsin \frac{g(x)}{2}$, what is h'(4) and why?

4. Why is the derivative of $\ln x$ equal to $\frac{1}{x}$?

5. Evaluate $\lim_{x\to\infty} \frac{x^2}{e^x}$

6. Why is the derivative of $\arctan x$ equal to $\frac{1}{1+x^2}$?

7. Bunny is a calculus student at Enormous State University, and she's having some trouble. Bunny says "OMG! This is literally impossible for anybody to ever pass! You learn something, but then the next day they tell you it's not right anymore! So, I got it when, like, the derivative of ln x was 1 over x, right? But then they had us do the derivative of ln 10, so I said 1 over 10, right? But they said it wasn't right anymore!"

Help Bunny understand what's going on.

- 8. A can of Mtn Dew contains 54 mg of caffeine. Suppose that amount enters the bloodstream immediately. If 40mg remains in the bloodstream 2 hours after drinking,
 - (a) Find a function of the form $f(x) = A b^x$ for the amount in the bloodstream after x hours have passed.

(b) What does your formula project the amount in the bloodstream will be after 5 hours?

(c) At what rate does your formula predict the amount of caffeine in the bloodstream will be changing after 5 hours?

9. Evaluate
$$\lim_{x \to 0} \frac{\sin^{-1} x}{\sqrt{x}}$$
.

10. Let $f(x) = \frac{1}{2-x}$. (a) Find $f^{-1}(x)$.

(b) Find f'(5) and $(f^{-1})'(-\frac{1}{3})$

(c) What's the connection between the answers to part b, and why?

Extra Credit (5 points possible): Evaluate $\lim_{x\to 0} \left(\frac{1}{\sin^2 x} - \frac{1}{x^2} \right)$