Exam 4 **Calc 1** **11/17/23**

Each problem is worth 10 points. For full credit provide good justification for your answers.

1. State the formal definition of the derivative of a function $f(x)$

2. Find the interval(s) on which $f(x) = 3x^2 + 6x - 5$ is decreasing.

3. Find the interval(s) where $f(x) = 2x^3 + 3x^2 - 36x$ is concave up.

4. Find the most general antiderivative of $f(x) = \sqrt[3]{x^2} + x\sqrt{x}$.

5. Use Newton's Method with the function $f(x) = x^2 - 3$ and initial value $x_0 = 2$ to calculate x_1 and x_2.

6. Find the absolute maximum and absolute minimum values of $f(x) = x\sqrt{9 - x^2}$ on $[1, 3]$

7. Biff is a calculus student at Enormous State University, and he's having some trouble. Biff says "Man, this calculus stuff is tough. We're doing this critical point stuff, right? And there was this one where there were two critical points, and I checked the first one and it was a max, right? So I figured the other one was automatically a min and I marked that on the answer sheet, but they said it was wrong. Isn't it, like, just a process of elimination like that?"

   Explain clearly to Biff what he can and can't conclude about a second critical point once he knows one critical point is a maximum.

8. [WW] A box is to be made out of a 12 cm by 18 cm piece of cardboard. Squares of side length x cm will be cut out of each corner, and then the ends and sides will be folded up to form a box with an open top. Find the dimensions of the box with the largest possible volume.

9. Find the dimensions of the largest rectangle that can fit in the first quadrant beneath $y = 12 - \frac{3}{2}x$.

10. For what values of the constant b does the function $f(x) = 2x^3 + bx^2 + 10x - 7$ have both a local maximum and local minimum point?

Extra Credit (5 points possible): Find the coordinates of all critical points of $y = \sin^3 x$ and identify them as maxima, minima, or inflection points.