Exam 3 **Calc 3** **12/1/23**

Each problem is worth 10 points. For full credit provide good justification for your answers.

1. Give a parametrization and bounds for t to produce a counterclockwise circle with radius 5 centered at the origin.

2. Let $\mathbf{F}(x, y, z) = \langle 3x, xy, xz \rangle$. Find div $\mathbf{F}$.

3. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F}(x, y) = y \sin x \,\mathbf{i} - \cos x \,\mathbf{j}$ and C is a line segment from $(0, 1)$ to $(0, 2)$.

4. Evaluate $\oint_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F}(x, y) = \langle 2xy^3 + y, 3x^2y^2 \rangle$ and C is a circle with radius 6 centered at the origin and traversed counterclockwise.

5. Let $\mathbf{F}$ be the vector field $\mathbf{F}(x, y, z) = 2x\mathbf{i} + 5y\mathbf{j} + 3z\mathbf{k}$. Let S be the sphere with radius 2 centered at the origin and oriented outward. Find $\iint_S \mathbf{F} \cdot d\mathbf{S}$.

6. Show that for any scalar function $f(x, y, z)$ with continuous second-order partial derivatives, $\text{curl}(\nabla f) = \mathbf{0}$. Make it clear how the requirement that the partials be continuous is important.

7. Biff is a calculus student at Enormous State University, and he's having some trouble. Biff says "Crap. This line intergul stuff is crazy. But I figured out that it's just like Calc 2 where lots of times you just flip the sign on your answer, right? So there was this one where they said the line intergul through the bottom half of a circle came out to 5, and they asked what it would be through the top half, so I said -5, and it was totally right! Why don't they just tell ya that's how it works?"

   Help Biff by explaining as clearly as you can whether his method will always work, and how you know.

8. Let $\mathbf{F}$ be the vector field $\mathbf{F}(x, y) = \langle P(x, y), 0\rangle$. Let C_1 be a differentiable curve where $y = f_1(x)$ from $(a, f_1(a))$ to $(b, f_1(b))$. Set up an integral for $\int_{C_1} \mathbf{F} \cdot dr$.

9. Let $\mathbf{F}$ be the vector field $\mathbf{F} = x\mathbf{i} - 10x\mathbf{j} + (-z - x)\mathbf{k}$. Let S be the bottom half of a sphere centered at the origin with radius 5, with normal vectors oriented outward. Find $\iint \text{curl } \mathbf{F} \cdot d\mathbf{S}$.

10. Let $\mathbf{F}(x, y) = \langle 0, 2y, 1 \rangle$, and let S be the portion of the paraboloid $z = x^2 + y^2$ below the plane $z = 4$ with upward orientation. Compute $\displaystyle\iint_S \mathbf{F} \cdot d\mathbf{S}$.

Extra Credit (5 points possible): If $\mathbf{a}$ is a constant vector, $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, and S is an oriented, smooth surface with simple, closed, smooth, positively oriented boundary curve C, show that

$$\iint_S 2\mathbf{a} \cdot d\mathbf{S} = \int_C (\mathbf{a} \times \mathbf{r}) \cdot d\mathbf{r}$$