

**Fake Quiz 1    Calc 3    11/18/2020**

This is a fake quiz, this is *only* a fake quiz. In the event of an actual quiz, you'd have been given fair warning. Repeat: This is *only* a fake quiz.

1. Compute  $\int_C (x^2 + y^2) dx - x dy$  along the quarter circle from (1,0) to (0,1).

Integrate the long way to get  $-1 - \pi/4$ .

2. Evaluate  $\iint_S \nabla \times \mathbf{F} \cdot \mathbf{n} d\sigma$  for the hemisphere  $S: x^2 + y^2 + z^2 = 9, z \geq 0$  and the field  $\mathbf{F} = y\mathbf{i} - x\mathbf{j}$ .

Use Stokes' Theorem to get  $-18\pi$ .

3. Evaluate  $\int (\sin y \sinh x + \cos y \cosh x) dx + (\cos y \cosh x - \sin y \sinh x) dy$  where  $C$  is the line segment from (1,0) to  $(2, \pi/2)$ .

Integrate using the Fundamental Theorem for Line Integrals (the potential function is  $f = \sin y \cosh x + \cos y \sinh x$ ) to get  $\cosh 2 - \sinh 1$ .

4. Evaluate  $\iint_S \mathbf{F} \cdot \mathbf{n} dS$ , where  $\mathbf{F}(x,y,z) = 4x\mathbf{i} - 3y\mathbf{j} + 7z\mathbf{k}$  and  $S$  is the surface of the cube bounded by the coordinate planes and the planes  $x = 1, y = 1,$  and  $z = 1$ .

Integrate using the Divergence Theorem to get 8.

5. Evaluate  $\iint_S \mathbf{F} \cdot \mathbf{n} dS$  where  $\mathbf{F}(x,y,z) = x\mathbf{i} + y\mathbf{j} + 2z\mathbf{k}$  and  $S$  is the portion of the cone  $z^2 = x^2 + y^2$  between the planes  $z = 1$  and  $z = 2$ , oriented upwards.

Integrate the long way to get  $14\pi/3$ .

6. Evaluate  $\int_C (x^2 - y) dx + x dy$ , where  $C$  is the circle  $x^2 + y^2 = 4$  with counterclockwise orientation.

Use Green's Theorem to get  $8\pi$ .

7. Evaluate  $\iint_S \langle x^3, x^2y, xy \rangle \cdot d\mathbf{S}$ , where  $S$  is the surface of the solid bounded by  $z = 4 - x^2, y + z = 5, z = 0,$  and  $y = 0$ .

Use the Divergence Theorem to get  $4608/35$ .

8. Compute  $\int_C \mathbf{F} \cdot d\mathbf{r}$  where  $\mathbf{F}(x,y,z) = y\mathbf{i} + z\mathbf{j} - x\mathbf{k}$  and  $C$  is the line segment from (1,1,1) to (-3,2,0).

Integrate the long way to get  $-13/2$ .

9. Compute  $\int_C \left\langle \ln(1+y), -\frac{xy}{1+y} \right\rangle \cdot d\mathbf{r}$  where  $C$  is the triangle with vertices (0,0), (2,0), and (0,4).

Use Green's Theorem to get  $-4$ .

10. Evaluate  $\int_{(0,1)}^{(\pi,-1)} y \sin x \, dx - \cos x \, dy$ .

Use the Fundamental Theorem for Line Integrals (the potential function is  $f = -y \cos x$ ) to get  $0$ .

11. Compute  $\iint_S \mathbf{F} \cdot \mathbf{n} \, dS$  where  $\mathbf{F}(x,y,z) = 2y \mathbf{j} + \mathbf{k}$  and  $S$  is the portion of the paraboloid  $z = x^2 + y^2$  below the plane  $z = 4$  with positive orientation.

Use the long way to get  $-12\pi$ .

12. Compute  $\int_C \mathbf{F} \cdot d\mathbf{r}$  where  $\mathbf{F}(x,y,z) = \langle 4x, 7y, -3z \rangle$  and  $C$  is the boundary of the first-octant portion of a sphere with radius  $5$  (centered at the origin).

Use Stokes' Theorem to conclude that, since  $\text{curl } \mathbf{F}$  is  $0$ , the surface integral (and hence the line integral) is  $0$ .