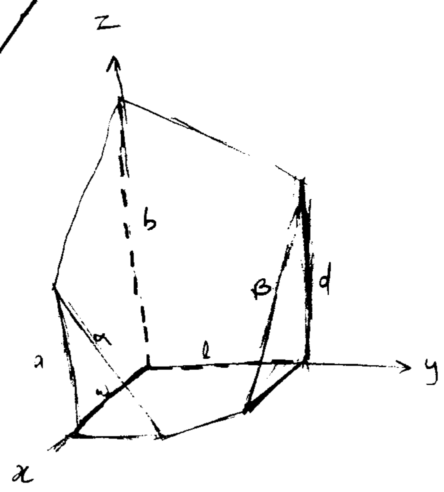


W/W

in cartesian co-ordinate system

Point on plane

$(0, 0, b), (0, l, d), (w, 0, a)$

Find two vectors on the plane

① $(0, l, d) - (0, 0, b)$

$= (0, l, d-b)$

② $(w, 0, a) - (0, 0, b)$

② $= (w, 0, a-b)$

normal vector given by cross-product of 1 × 2

$\langle 0, l, d-b\rangle \times \langle w, 0, a-b\rangle$

$$\begin{vmatrix} x & y & z \\ 0 & l & d-b \\ w & 0 & a-b \end{vmatrix}$$

$\vec{n} = (l(a-b)\,i + w(d-b)\,j + 0z) - (0x + 0y + wlz)k$

$\vec{n} = l(a-b)\vec{x} + w(d-b)\vec{y} - wl\vec{k}$

$\vec{u} = (x, y, z-b)$

∴ Equation of the plane is $\vec{n}\cdot\vec{u} = 0$

$Eqn = l(a-b)x + w(d-b)y + (z-b)(-wl) = 0$

$l(a-b)x + w(d-b)y - wlz + wlb = 0$

$= \underline{\underline{l(a-b)x + w(d-b)y}}$

Good

Equation of bottom plane

$\vec{n} = 0, 0, 1.$

$\vec{u} = x, y, z$

$Eqn = \underline{z = 0}$

Intersecting line @

$l(a-b)x + w(d-b)y - wlz + wlb = z = 0$

$l(a-b)x + w(d-b)y + wlb = 0$

$y = \dfrac{l(a-b)x + wlb}{w(b-d)} \quad \Big| \quad \forall x < w \text{ and } y < l$

Nice

when $x = w$ on the surface of the top plane
the line α is revealed

and when plugged in

$$l\cancel{w}(a-b) + \cancel{w}(d-b)y - \cancel{w}lz + \cancel{w}lb = 0$$

then this line crosses the $z=0$ plane gives where α crosses the xy-plane

$$la - \cancel{lb} + dy - by + \cancel{lb} = 0$$

$$la + (d-b)y = 0$$

$$y = \frac{-la}{d-b} = \frac{la}{b-d} \quad \therefore \text{ point} = \left(w, \frac{la}{b-d}, 0\right)$$

When $y = l$ on the top surface(plane) the line B is revealed

$$\cancel{l}(a-b)x + w(d-b)\cancel{l} - w\cancel{l}z + w\cancel{l}b = 0$$

for $z=0$

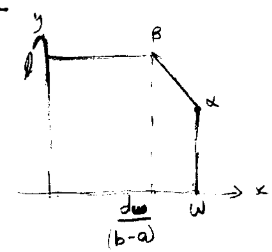
$$ax - bx + dw - \cancel{bw} + \cancel{wb} = 0$$

$$(a-b)x = -dw$$

$$x = \frac{dw}{b-a}$$

other point B x-y plane intercept $\left(\frac{dw}{b-a}, l, 0\right)$

Top view



Equation of plane =

$$y = \frac{l(a-b)x + wlb}{w(b-d)}$$

$$z = \frac{l(a-b)x + w(d-b)y + wlb}{wl}$$

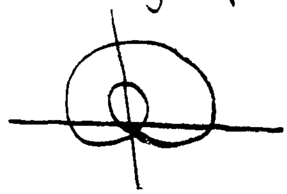
$$\text{Volume} = \frac{1}{wl}\int_0^{\frac{dw}{b-a}}\int_0^{l} \left(l(a-b)x + w(d-b)y + wlb\right) dy\,dx$$

$$+ \frac{1}{wl}\int_{\frac{dw}{b-a}}^{w}\int_0^{\frac{l(a-b)x + wlb}{w(b-d)}} \left(l(a-b)x + w(d-b)y + wlb\right) dy\,dx$$

Excellent

10/10

2) The general form $a+b\sin\theta$ where $a<b$ produces a graph like this:



We are interested in finding the center of mass of this object. We can find the values of θ ~~that~~ form the outer loop by setting the equation equal to zero.

$$0 = a + b\sin\theta$$

$$\sin^{-1}\left(\frac{-a}{b}\right) = \theta$$

10

So the integral for the outer loop will have the general form:

$$\int_{\sin^{-1}\left(-\frac{a}{b}\right)}^{\pi-\sin^{-1}\left(-\frac{a}{b}\right)} \int_{0}^{a+b\sin\theta} \quad r\,dr\,d\theta$$

And, the integral of the inner loop will have the general form:

$$\int_{\pi-\sin^{-1}\left(-\frac{a}{b}\right)}^{\sin^{-1}\left(-\frac{a}{b}\right)} \int_{0}^{a+b\sin\theta} \quad r\,dr\,d\theta$$

Now, the question is do we add or subtract the smaller from the bigger!

Excellent

We can answer this by using a specific form

$$r = 1 + 2\sin\theta$$

and taking the iterated integral of the smaller loop to find its area. If it results in a positive value, then we know the inner loop must be subtracted from the outer loop

$$\int_{7\pi/6}^{11\pi/6} \int_0^{1+2\sin\theta} r\,dr\,d\theta = 3.11$$

This is positive, so we must subtract the smaller loop from the bigger loop.

$$m = \int_{\sin^{-1}(\frac{a}{b})}^{\pi - \sin^{-1}(\frac{a}{b})} \int_0^{a+b\sin\theta} Kr\,dr\,d\theta - \int_{\pi-\sin^{-1}(\frac{a}{b})}^{\sin^{-1}(\frac{a}{b})} \int_0^{a+b\sin\theta} Kr\,dr\,\theta$$

$$M_y = \int_{\sin^{-1}(\frac{a}{b})}^{\pi - \sin^{-1}(\frac{a}{b})} \int_0^{a+b\sin\theta} Kr^2\cos\theta\,dr\,d\theta - \int_{\pi-\sin^{-1}(\frac{a}{b})}^{\sin^{-1}(\frac{a}{b})} \int_0^{a+b\sin\theta} Kr^2\cos\theta\,dr\,d\theta$$

$$M_x = \int_{\sin^{-1}(\frac{a}{b})}^{\pi - \sin^{-1}(\frac{a}{b})} \int_0^{a+b\sin\theta} Kr^2\sin\theta\,dr\,d\theta - \int_{\pi-\sin^{-1}(\frac{a}{b})}^{\sin^{-1}(\frac{a}{b})} \int_0^{a+b\sin\theta} Kr^2\sin\theta\,dr\,d\theta$$

$$\boxed{\bar{x} = \frac{M_y}{m} = \frac{\int_{\sin^{-1}(\frac{a}{b})}^{\pi - \sin^{-1}(\frac{a}{b})} \int_0^{a+b\sin\theta} Kr^2\cos\theta\,dr\,d\theta - \int_{\pi-\sin^{-1}(\frac{a}{b})}^{\sin^{-1}(\frac{a}{b})} \int_0^{a+b\sin\theta} Kr^2\cos\theta\,dr\,d\theta}{\int_{\sin^{-1}(\frac{a}{b})}^{\pi - \sin^{-1}(\frac{a}{b})} \int_0^{a+b\sin\theta} Kr\,dr\,d\theta - \int_{\pi-\sin^{-1}(\frac{a}{b})}^{\sin^{-1}(\frac{a}{b})} \int_0^{a+b\sin\theta} Kr\,dr\,d\theta}}$$

$$\boxed{\bar{y} = \frac{M_x}{m} = \frac{\int_{\sin^{-1}(\frac{a}{b})}^{\pi - \sin^{-1}(\frac{a}{b})} \int_0^{a+b\sin\theta} Kr^2\sin\theta\,dr\,d\theta - \int_{\pi-\sin^{-1}(\frac{a}{b})}^{\sin^{-1}(\frac{a}{b})} \int_0^{a+b\sin\theta} Kr^2\sin\theta\,dr\,d\theta}{\int_{\sin^{-1}(\frac{a}{b})}^{\pi - \sin^{-1}(\frac{a}{b})} \int_0^{a+b\sin\theta} Kr\,dr\,d\theta - \int_{\pi-\sin^{-1}(\frac{a}{b})}^{\sin^{-1}(\frac{a}{b})} \int_0^{a+b\sin\theta} Kr\,dr\,d\theta}}$$