$\frac{20}{20}$

Problem Set ~~2~~ 3

1) The paraboloid $z=x^2+y^2$ is centered around the z-axis and facing up, sitting on the origin. The paraboloid $z=9-(x+1)^2-y^2$ is facing down, moved back on the x-axis 1 and up on the z-axis 9. So the z limits are from x^2+y^2 to $9-(x+1)^2-y^2$. Now we need to find where these two intercept.

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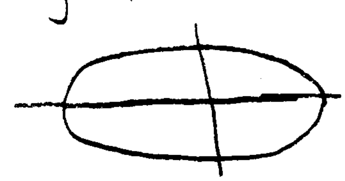
$$x^2+y^2=9-(x+1)^2-y^2$$
$$2y^2=9-(x+1)^2-x^2$$
$$y^2=\frac{9}{2}-\frac{(x+1)^2}{2}-\frac{x^2}{2}$$
$$y=\pm\sqrt{\frac{9}{2}-\frac{(x+1)^2}{2}-\frac{x^2}{2}}$$

We graph this to see the top view:



So, the y-limits are $-\sqrt{\frac{9}{2}-\frac{(x+1)^2}{2}-\frac{x^2}{2}}$ to $\sqrt{\frac{9}{2}-\frac{(x+1)^2}{2}-\frac{x^2}{2}}$ and to find the x-limits we need to solve for $y=0$.

$$0=\frac{9}{2}-\frac{x^2+2x+1}{2}-\frac{x^2}{2}$$
$$\frac{9}{2}=\frac{2x^2+2x+1}{2}$$
$$2x^2+2x-8$$
$$x=1.56 \quad x=-2.56$$

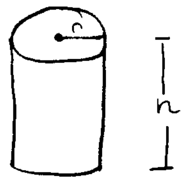
So, $\left(\frac{-1+\sqrt{17}}{2}\right)$ the integral is:

$$\int_{-2.56}^{1.56} \int_{-\sqrt{\frac{9}{2}-\frac{(x+1)^2}{2}-\frac{x^2}{2}}}^{\sqrt{\frac{9}{2}-\frac{(x+1)^2}{2}-\frac{x^2}{2}}} \int_{x^2+y^2}^{9-(x+1)^2-y^2} 1 \, dz\,dy\,dx$$

$\left(\frac{-1-\sqrt{17}}{2}\right)$

Using Mathematica: $\underline{V = 56.745}$

2) a)



$V_{cylinder} = \pi r^2 h$

$\iiint dV$

$\int_0^{2\pi}\int_0^{h}\int_0^{r} r\,dr\,dz\,d\theta$

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$= \int_0^{2\pi}\int_0^{h} \left[\tfrac{1}{2} r^2\right]_0^r dz\,d\theta$

$= \int_0^{2\pi}\int_0^{h} \left[\tfrac{1}{2} r^2\right] dz\,d\theta$

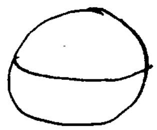
$= \int_0^{2\pi} \left[\tfrac{1}{2} r^2 h\right]_0^h d\theta$

$= \int_0^{2\pi} \tfrac{1}{2} r^2 h\, d\theta$

$= \tfrac{1}{2} r^2 h\, \theta \Big|_0^{2\pi}$

$= 2\pi\, \tfrac{1}{2} r^2 h$

$= \underline{\pi r^2 h}$

b) 

$V_{sphere} = \frac{4}{3}\pi r^3$

$$\iiint dV$$

$$\int_0^{2\pi}\int_0^{\pi}\int_0^{r} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

$$\int_0^{2\pi}\int_0^{\pi} \tfrac{1}{3}\rho^3 \sin\phi \Big|_0^r \, d\phi \, d\theta$$

$$= \int_0^{2\pi}\int_0^{\pi} \tfrac{1}{3} r^3 \sin\phi \, d\phi \, d\theta$$

$$= \int_0^{2\pi} \tfrac{1}{3} r^3 \cdot -\cos\phi \Big|_0^{\pi} \, d\theta$$

$$= \int_0^{2\pi} \tfrac{1}{3} r^3 \cdot -((-1)-1) \, d\theta$$

$$= \int_0^{2\pi} \tfrac{2}{3} r^3 \, d\theta$$

$$= \tfrac{2}{3} r^3 \theta \Big|_0^{2\pi}$$

$$= 2\pi \tfrac{2}{3} r^3$$

$$= \underline{\underline{\tfrac{4}{3}\pi r^3}}$$