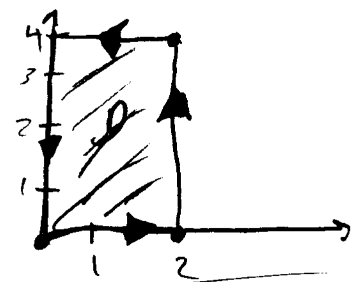
Calculus IV　　Quiz 5　　Fall 1999　　11/15/99

Green's Theorem (provided the proper conditions apply):

$$\oint_C P\,dx + Q\,dy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA$$

1. Use Green's Theorem to compute $\oint_C (x^2+y^2)\,dx - xy^2\,dy$ where C is the positively oriented rectangle having vertices (0,0), (2,0), (2,4), and (0,4).



$\oint_C (x^2+y^2)dx - xy^2dy = \iint_D (-y^2-2y)\,dA$

since $\frac{\partial Q}{\partial x} = -y^2$, $\frac{\partial P}{\partial y} = 2y$

$= \int_0^2\int_0^4 -y^2-2y\ dy\,dx = -\int_0^2 \left[\frac{y^3}{3}+y^2\right]_0^4 dx$

$= -\int_0^2 \frac{64}{3}+16\ dx = -\int_0^2 \frac{64}{3}+\frac{48}{3}\,dx = -\int_0^2 \frac{112}{3}\,dx = \frac{-224}{3}$

2. Compute the curl of the vector field $\mathbf{F}(x,y,z) = x^2\mathbf{i} - yz\mathbf{j} + (x+y+z)\mathbf{k}$.

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curl is cross prod

$$\left|\begin{array}{ccc|cc} i & j & k & i & j \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} & \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \\ x^2 & -yz & (x+y+z) & x^2 & -yz \end{array}\right|$$

left - right

$$= (1i + 0j + 0k) - (0k + -yi + 1j)$$

$$= 1i + yi + 1j + 0k$$

$$= \underline{(1+y)i + 1j + 0k}$$

3. Compute the divergence of the vector field $\mathbf{F}(x,y,z) = x^2\mathbf{i} - yz\mathbf{j} + (x+y+z)\mathbf{k}$.

$$\partial/\partial x \; x^2 + \partial/\partial y \; -yz + \partial/\partial z \; (x+y+z)$$

$$= 2x - z + 1$$

$$\boxed{\text{div } F = 2x - z + 1}$$