Practice Exam 1 Solutions Algebra & Trig 2/26/2003

Each problem is worth 10 points. For full credit provide complete justification for your answers.

Warning: Only answers are given here. These alone would probably get at most half credit on the exam. Justification counts, but the answers are provided here to let you know if you're on the right track or not.

1. Simplify $5y - 2y[7 - 3(y - 2)]$.

$6y^2 - 21y$

2. Solve $5x - 9 = 3x + 7$.

$x=8$

3. Rewrite $-5 < x \leq 2$ both in interval notation and on a number line.

(-5,2]

The number line should look something like:

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4. Simplify $(8x^{12}y^{-6})^{1/3}$ and write the answer using positive exponents only.

$$\frac{2x^4}{y^2}$$

5. Solve $2x - 3y = 7$
$3x - y = 7$

$x = 2,\ y = -1$

6. Write $\dfrac{3+4i}{2-i}$ in standard form.

$$\frac{2}{5}+\frac{11}{5}i$$

7. Solve $|2x + 1| < 5$ and express the solution both with interval notation and on a number line.

(-3,2)

The number line should look something like:

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8. Solve $\sqrt{2w + 5} - w = 1$.

$w = 2$

9. Buzz is a precalc student at the University of Iowa who's having some trouble. He says "Whoa, man, this math class is kicking my butt. There was this question on our test that was, like, $x^3 + x^2 - 5x + 3 = 0$, and we were supposed to tell if $x = 1$ was a right answer for it. But, like, I never saw in High School how you solve ones where there's x^3 in it, so I had no chance at all!"

Explain to Buzz, clearly enough that he can understand, how he could have answered the question even without knowing how to solve a third degree equation.

There are lots of ways to answer that would be good, but one possibility would be something like:

Okay, Buzz, listen a minute. You didn't have to know how to solve that equation, you just need to know how to tell if something works as a solution. Being a solution means being a number that works when you put it in the equation, so just try it. If you put 1 in the x spot you have $(1)^3 + (1)^2 - 5(1) + 3 = 0$, which is $1 + 1 - 5 + 3 = 0$, and that's true. So that means 1 works as a solution.

If it hadn't worked, you could tell that too because you'd get something that wasn't true. Think about if you tried $x = 2$, you'd have $(2)^3 + (2)^2 - 5(2) + 3 = 0$, or $8 + 4 - 10 + 3 = 0$, and that's not true, so $x = 2$ isn't a solution to the equation.

10. Consider equations of the form $x^2 - 2x = c$. For what values of c will such an equation have real solutions?

We can rearrange the equation to put it in the usual form $x^2 - 2x - c = 0$, and then we can use the quadratic formula on it to get $x = \dfrac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(c)}}{2(1)}$. What matters for having real solutions is whether the stuff under the radical is positive, negative, or zero, so we can just look at it. It simplifies to $4 - 4c$, so if $4 - 4c \geq 0$, we have $c \leq 1$.

So the equation will have (at least one) real solutions when $c \leq 1$, and only imaginary solutions otherwise. Notice that what's hard about this problem isn't the actual work that you do – it's recognizing that's what needs to be done.

Extra Credit (5 points possible): An earthquake emits a primary wave and a secondary wave. Near the surface of the Earth the primary wave travels at about 5 miles per second, and the secondary wave travels about 3 miles per second. From the time lag between the two waves arriving at a given seismic station, it is possible to estimate the distance to the quake. Suppose a station measures a time difference of 16 seconds between the arrival of the two waves. How far is the earthquake from the station?

120 miles