Practice Exam 2 Key   Algebra & Trig   3/17/2003

Each problem is worth 10 points. For full credit provide complete justification for your answers.

Notice that generally answers alone are given here, and that without more justification these would generally receive at most half credit.

1. Find the center and radius of the circle with equation $x^2 + y^2 - 2x - 10y = 55$ and sketch the circle.

Completing the square makes the equation $(x-1)^2 + (y-5)^2 = 81$ so the center is (1,5) and the radius is 9. Your sketch should include at least the fact that (10,5), (-8,5), (1,14) and (1, -4) will be points on the circle.

2. Find the slope and both x- and y-intercepts of the line with equation $4x - 5y = 10$.

The slope is 4/5, the y-intercept is at -2, and the x- intercept is at 5/2.

3. Find an equation for the line through the point (3,9) with slope $m = 6$.

$y - 9 = 6(x - 3)$, which can be simplified if it'll make you feel better.

4. If $f(x) = x^3$ and $g(x) = x - 2$, find:

$f(2) = 8$

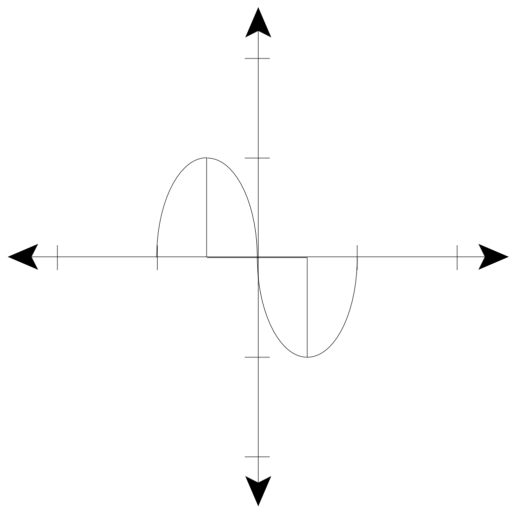
$g(7) = 5$

$\left(\frac{f}{g}\right)(3) = 27$

$f(2 + h) = (2 + h)^3$ (which you can multiply out if it makes you feel better)

$g(x + h) = (x + h) - 2$ (with or without the parentheses)

5. If f(x) has the graph shown below, sketch graphs of f(x) + 1, f(x-1), f(-x), and 2f(x).



They'll be, respectively, the same graph moved 1 unit higher, the same graph moved one unit to the right, the same graph reflected across the y-axis, and the same graph but stretched twice as far away from the x-axis.

6. Find all zeros of the polynomial $p(x) = 3x^3 - 2x^2 - 3x + 2$.

1, -1, and 2/3

7. Factor the polynomial $2x^3 - 9x^2 + 2x + 20$ completely.

$$(2x-5)\left(x-\left(1+\sqrt{5}\right)\right)\left(x-\left(1-\sqrt{5}\right)\right)$$

8. Decompose $\dfrac{5x^2+2x+9}{x^4-3x^3+x^2-3x}$ into partial fractions.

$$\frac{-3}{x}+\frac{2}{x-3}+\frac{x-1}{x^2+1}$$

9. Polly is a Precalc student at Enormous State University, and she's having some trouble with asymptotes. Polly says "Okay, so, like, those rational thingys are totally making me crazy. There was this problem on the practice test, and it told you all this stuff, like, if you know that a rational function has vertical asymptotes at $x = -3$ and $x = 1$, a horizontal asymptote at $y = 0$, and that it crosses the x axis only when $x = 0$, then what are the possible formulas for that function? But, like, it makes it sound like there could be lots of them, and I don't even know how there could be, or how I could find even one of them. And the teacher doesn't speak English hardly at all, and when I went to his office to ask for help he yelled at me and slammed the door in my face. I'm so totally gonna fail!"

Explain to Polly as clearly as possible how you could know what a formula for such a function would be, and how there could be more than one.

Since there are vertical asymptotes at $x = -3$ and $x = 1$, the denominator must have terms of $(x+3)$ and $(x-1)$. Also since the graph crosses the x-axis at $x = 0$, the numerator must have a factor of $(x-0)$ or just x. You also know that the degree of the numerator must be less than the degree of the denominator, since there is a horizontal asymptote at $y = 0$. This leaves lots of things you don't know, though, like that the $(x+3)$ and $(x-1)$ in the denominator could have higher multiplicity than just 1, as could the x in the numerator, and there could be constants or even other factors of the form (x^2+1), say, since they wouldn't lead to other asymptotes or intercepts, so there are lots of possibilities other than just the basic $f(x) = \dfrac{x}{(x+3)(x-1)}$.

10. For what values of b does $f(x) = \dfrac{x^3 + bx^2 + 3x}{x^2 + 2}$ have exactly two x-intercepts?

The x-intercepts happen when the numerator is 0, so you want to think about solutions of $x^3 + bx^2 + 3x = 0$. Once you factor an x out what's left is $x(x^2 + bx + 3) = 0$, which will have one solution when $x = 0$ and others which come from the solutions of $x^2 + bx + 3 = 0$, which we can use the quadratic formula on. For there to be exactly two solutions, since we already know what one of them is (the $x = 0$ one), the quadratic formula will have to produce exactly one more solution, which happens when the stuff under the radical in the quadratic formula is 0. So we want values of b for which $b^2 - 12 = 0$, or $b = \pm\sqrt{12}$.

Extra Credit (5 points possible): If you were asked what $\sqrt[3]{1}$ is, you'd probably just say 1. However, the equation $x^3 = 1$ has two other solutions as well. Find the other two.

$$\frac{-1}{2} \pm \frac{\sqrt{3}}{2} i$$