Exam 3 Algebra & Trig 4/25/2003

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. Convert $\frac{\pi}{10}$ to an equivalent exact degree measure.

$$\frac{\pi}{10} \cdot \frac{180}{\pi} = \frac{180\pi}{10\pi} = \boxed{18^\circ}$$

Good

2. Give an approximation (accurate to at least three decimal places) for $\log_2 7$.

$\log_3 32$

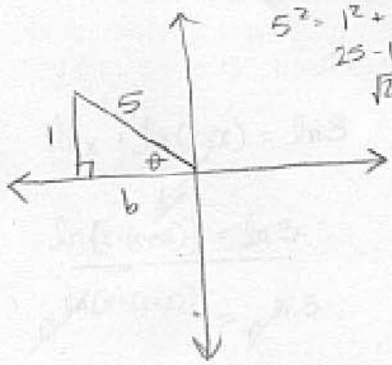
$$\log 2^x = \log 7$$

$$x = \frac{\log 7}{\log 2}$$

Great

$$\boxed{x = 2.807}$$

3. If θ is a second-quadrant angle such that sin θ = 1/5, find exact values for the other 5 trig functions of θ.



$5^2 = 1^2 + b^2$

$25 - 1 = b^2$

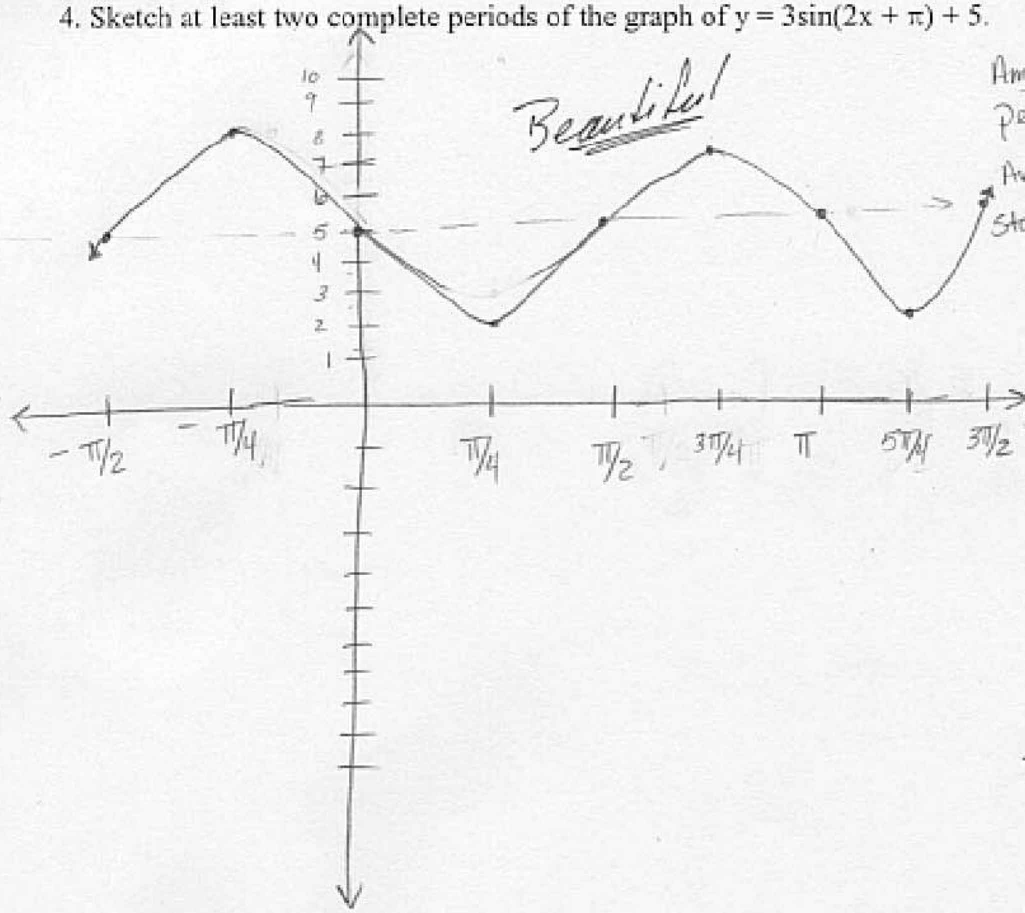
$\sqrt{24} = \sqrt{b^2}$

$\sqrt{24} = b$

$\sin\theta = \frac{1}{5}$	$\csc\theta = 5$
$\cos\theta = -\frac{\sqrt{24}}{5}$	$\sec\theta = -\frac{5}{\sqrt{24}}$
$\tan\theta = -\frac{1}{\sqrt{24}}$	$\cot\theta = -\sqrt{24}$

Great

4. Sketch at least two complete periods of the graph of y = 3sin(2x + π) + 5.



Beautiful

Amp = 3

Period = $\frac{2\pi}{2} = \pi$

Aver = 5

Starts: $2x + \pi = 0$

$\frac{2x}{2} = -\frac{\pi}{2}$

$x = -\frac{\pi}{2}$

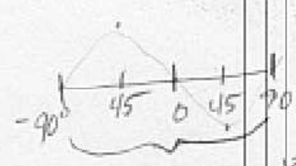
Great!

Ends: $2x + \pi = 2\pi$

$2x = 2\pi - \pi$

$\frac{2x}{2} = \frac{\pi}{2}$

$x = \frac{\pi}{2}$



$\frac{45}{1} \cdot \frac{\pi}{180} = \frac{\pi}{4}$

$\frac{270°}{1} \cdot \frac{\pi}{180} = 3\pi/2$

$\frac{135°}{1} = \frac{\pi}{180} = 3\pi/4$

$\frac{225°}{1} = \frac{\pi}{180} = 5\pi/4$

5. Solve (exactly) the equation $\ln x + \ln (x - 2) = \ln 3$.

$$\ln x(x-2) = \ln 3$$

$$e^{\ln x(x-2)} = e^{\ln 3}$$

$$x(x-2) = 3$$

$$x^2 - 2x - 3 = 0$$

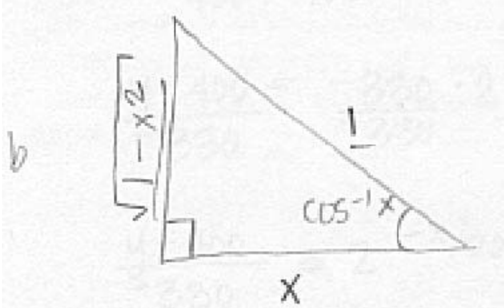
$$(x-3)(x+1) = 0$$

$$\boxed{x = 3} + \cancel{x = -1}$$

Good

7. Simplify $\tan(\cos^{-1} x)$.

SOH CAH TOA



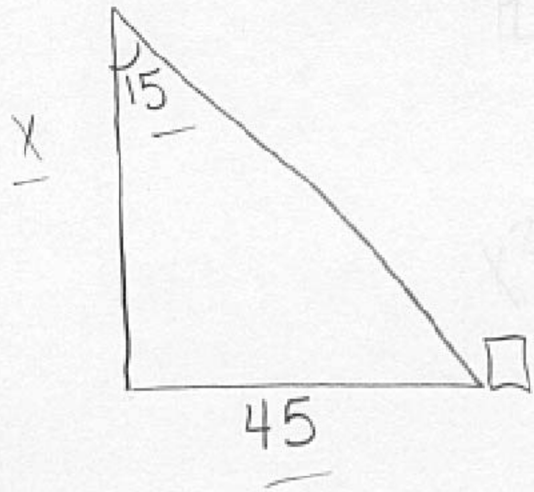
$$\frac{x}{1} = \frac{A}{H}$$

$$\sqrt{1^2 - x^2} = b$$

Great

$$\boxed{\tan = \frac{\sqrt{1-x^2}}{x}}$$

6. A devious hotdog vendor notices a bored office worker measuring the angle from his boss's window to his hotdog stand, and carefully repositions the stand while the office worker is on his way down the elevator to get lunch. If the hotdog stand was actually 45 feet away from the base of the building when the worker measured the angle between the vertical side of the building and the stand to be 15°, how tall (to the nearest foot) is the window?



$$\tan 15 = \frac{45}{X}$$

$$\tan 15 X = 45$$

$$X = \frac{45}{\tan 15}$$

$X = 168$ feet

Excellent

8. The temperature after x minutes of a yam placed in an oven is given by $f(x) = 400 - 330 \cdot 2^{-x/20}$. Find $f^{-1}(x)$.

$$y = 400 - 330 \cdot 2^{-x/20}$$

$$y - 400 = -330 \cdot 2^{-x/20}$$

$$\frac{y-400}{-330} = 2^{-x/20}$$

$$\log_2\left(\frac{y-400}{-330}\right) = \log_2 2^{-x/20}$$

$$\log_2\left(\frac{y-400}{-330}\right) = -x/20$$

$$-20 \log_2\left(\frac{y-400}{-330}\right) = x$$

Well done!

$$\boxed{f^{-1}(x) = -20 \log_2\left(\frac{x-400}{-330}\right)}$$

9. Paula is a Precalc student at Enormous State University, and she's having some trouble with trigonometry. She says "I just so totally don't get all this stuff about graphs of trig functions. I figured I could pretty much just know nothing since my calculator does it all, but I totally screwed up on the test, and now the professor is saying since everybody else did bad too he's probably gonna put that on the final too. There was this problem where they gave this graph of, like, one of those up-and-down trig functions, and we were supposed to figure out a formula for it, and my calculator can't do it that way at all. So anyway, I thought it must be a cosine one, because it started out at the lowest spot, but my friend said it had to be a sine one. How do you know which it has to be?"

Explain clearly to Paula how to know if a graph is of a sine function, a cosine function, or both.

The graph can be not only cosine one

but also a sine one.

The formula will either $y = A\cos(x - c) + D$

or $y = A\sin(x - C) + D$

Then

$$\left[y = A\cos(x - c) + D\right] = \left[y = A\sin\left(x - c + \frac{\pi}{2}\right) + D\right]$$

If you change "C" yor formula

can be both cosine one and sine one.

Exactly!

10. If the amount of Carbon-14 (which undergoes radioactive decay at a rate which leaves half the original amount after approximately 5600 years) in a yak fossil was originally 3mg, and is now 0.1mg, how old (to the nearest hundred years) is the fossil?

$$C = C_0 \times \frac{1}{2}^{\frac{t}{5600}}$$

let $C = 0.1$

$C_0 = 3$

$$0.1 = 3 \times \frac{1}{2}^{\frac{t}{5600}}$$

Yes

$$\frac{0.1}{3} = \frac{1}{2}^{\frac{t}{5600}}$$

$$\frac{\ln \frac{0.1}{3}}{\ln \frac{1}{2}} = \frac{t}{5600}$$

$$t = 5600 \times \frac{\ln \frac{0.1}{3}}{\ln \frac{1}{2}}$$

$$= 27478$$

27500 years

Great Job!