Exam 2b   Calc 2   3/21/2003

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. **Write an integral** for the length of the curve y = x^2 between (0,0) and (3,9).

2. **Integrate** $\int x\cot^2\left(x^2\right)dx$ [Hint: Our table of integrals says $\int \cot^2 u\,du = -\cot u - u + C$].

3. **Show** that $\int_0^{\infty} x \cdot e^{-x}\, dx = 1$.

4. Line 65 in our Table of Integrals says $\int \tan^2 u\, du = \tan u - u + C$. Suppose someone tells you that they suspect there's a typographical error there, because integrals should raise the degree of things rather than decrease it. **Verify** that the table entry is correct or erroneous.

5. **Integrate** $\int \frac{1}{9-x^2}\,dx$ [Hint: The answer can be written $\frac{-1}{6}\ln\left|\frac{x-3}{x+3}\right| + C$].

6. Derive the formula $\int \sqrt{2au-u^2}\,du = \frac{u-a}{2}\sqrt{2au-u^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{u-a}{a}\right) + C$ [You're welcome to use the integral formula $\int \sqrt{a^2-u^2}\,du = \frac{u}{2}\sqrt{a^2-u^2} + \frac{a^2}{2}\sin^{-1}\frac{u}{a} + C$ if you find it convenient].

7. **Find** the x coordinate of the centroid of the trapezoidal region with vertices at (0,0), (a,0), (0,b), and (a,c).

8. **Find** the surface area of the cone obtained by rotating the segment of the line $y = \frac{r}{h}x$ on [0,h] around the x-axis.

9. Brandi is a calculus student at E.S.U. who's having some trouble with improper integrals. Brandi says "So there was this problem on our test, and it was to say if $\int_1^\infty \frac{1}{x \ln x} dx$ converged or not. I drew this total blank on how to find the antiderivative, but I thought about it and decided I didn't really have to. See, you know that if you integrate 1/x it's infinity, but if you integrate $1/x^2$ or anything else where the denominator is more than just x it converges. So I just said all that and said it must converge. But the grader gave me no points at all, and just said that didn't work, but not why, and then he told me to go away because he hates dealing with students."

Explain to Brandi either why she's right, or what's wrong with her reasoning.

10. Given that $\int_0^{\infty} e^{-x^2}\, dx = \frac{1}{2}\sqrt{\pi}$, **find** Γ(½). [Remember that Γ(x) = $\int_0^{\infty} t^{x-1} e^{-t}\, dt$].

Extra Credit (5 points possible): Find the area of the region under the curve y = $1/(1+e^x)$ over the interval [-ln 5, ln 5].