**Problem Set 1        Calc 2        Due 2/14/2003**

Each problem is worth 5 points. For full credit provide complete justification for your answers.

1. If $[\![x]\!]$ denotes the greatest integer less than or equal to x,

   (a) evaluate $\int_0^n [\![x]\!]\, dx$ , where $n$ is a positive integer.

   (b) evaluate $\int_m^n [\![x]\!]\, dx$ , where $m$ and $n$ are both positive integers with $m \leq n$.

2. (a) Find the area of the region bounded by the parabola $y = x^2$, the tangent to this parabola at (1,1), and the x axis.

   (b) Generalize your answer to part (a) by finding the area of the region bounded by the parabola $y = x^2$, the tangent line to this parabola at $(c,c^2)$, and the x axis, for any given value of $c$.

3. Pat the mathematician is going to go into business making napkin rings. Pat plans to start with wooden spheres, and then drill a hole through the center of each so that the height of the portion remaining is $h$. If a sphere starts out with a radius of $R$, find out the volume of wood remaining in the finished napkin ring.

4. [Inspired by Judith V. Grabiner's "'Some Disputes of Consequence': Maclaurin among the Molasses Barrels," from *Social Studies of Science* 28/1 (February 1998) 139-68] In 1735 the great British mathematician Colin Maclaurin "wrote a 94-page memoir to the Scottish Excise Commission, explaining how to gauge, with a single dip of a dipstick, the amount of molasses in the barrels in the Port of Glasgow." [p. 139] In this treatise he proved several surprising theorems to the general effect that the difference between the frustum of the solid produced by revolving a conic section around one of its axes and an approximating cylinder matching the radius at the midpoint of the frustum depends only on the height of the frustum. In particular, he proved that for a paraboloid of revolution the volume of the frustum is the same as the volume of the cylinder. Use a double integral to express the volume of a frustum of a paraboloid of revolution and show why this is true.