

Problem Set 3 Calc 2 Due 3/14/2003

Each problem is worth 5 points. For full credit provide complete justification for your answers.

1. Integrate $\int \frac{x^2 + x - 2}{3x^3 - x^2 + 3x - 1} dx$.

2. [Based on Stewart 4th 8.1 #40] The curves with equations $x^n + y^n = 1$ for even values of $n \geq 4$ are called **fat circles**. Graph several of these curves to understand why. Set up an integral for the length L_{2k} of the fat circle with $n = 2k$. Approximate L_4 and L_6 correct to at least 4 decimal places. What is the value of $\lim_{k \rightarrow \infty} L_{2k}$?

3. The **Gamma function**, $\Gamma(x)$, is defined by the formula $\Gamma(x) = \int_0^{\infty} t^{x-1} e^{-t} dt$. Find $\Gamma(1)$, $\Gamma(2)$, $\Gamma(3)$, $\Gamma(4)$, and $\Gamma(5)$. Explain in words what the gamma function does with whole-number inputs.

4. The surface obtained by rotating the portion of $y = \frac{1}{x}$ on the interval $[1, \infty)$ around the x axis is known as **Gabriel's Horn**. Find the volume and surface area of Gabriel's Horn.