Exam 1 Differential Equations 3/7/2003

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. **Show** that $y(t) = e^{-t} + t^2$ is a solution to the differential equation $\frac{dy}{dt} + y = t^2 + 2t$.

2. Given that $S(t) = \frac{30t + t^2 + C}{15 + t}$ is a general solution to a differential equation, **find a particular solution** which satisfies the initial condition $S(0) = 3$.

3. **Give an example** of a differential equation which is not linear.

4. An enormous vat begins with 50 gallons of pure water at time $t = 0$. More pure water is added from one spigot at the rate of 5 gallons per minute. Meanwhile, a second spigot adds salt water with a concentration of 0.5 pounds of salt per gallon at a rate of 4 gallons per minute. The solution is kept well mixed, and 3 gallons per minute are drained out from the bottom of the tank. **Write a differential equation** for the rate of change of the amount of salt in the tank over time.

5. **Sketch the phase line** for the differential equation $\frac{dy}{dt} = 0.05t(t-1)(t-8)$ and label each equilibrium point as a sink, source, or node.

6. Consider the differential equation $\frac{dP}{dt} = 2P - \frac{P^2}{50} - 30$ for the population of fish in a lake if 10 fishing licenses are granted, with initial condition $P_0 = 100$. **Use Euler's Method** (with a step size of $\Delta t = 1$) to approximate the population at $t = 1$ and $t = 2$.

7. A yam is placed in a hot oven at time $t = 0$ and begins to warm up according to the differential equation $\frac{dT}{dt} = k(400 - T)$. **Find a general solution** $T(t)$ for the temperature of the yam after t minutes in the oven.

8. **Sketch the bifurcation diagram** for the differential equation $\frac{dy}{dt} = ay - y^3$, where a is the parameter.

9. The differential equation $2ty\dfrac{dy}{dt} = 4t^2 + 3y^2$ is not separable. Use the substitution $u = \dfrac{y}{t}$ to transform it into a new differential equation which is separable, and make it clear how you know it is separable. **You do not need to solve the new equation.**

10. The amount of salt in a polluted pond is modeled by the differential equation $\frac{dS}{dt} = \frac{5}{2} - \frac{26S}{200 - t}$. **Find a general solution** to this differential equation.