Exam 2 Differential Equations 4/11/2003

Each problem is worth 10 points. For full credit provide complete justification for your answers.

Use the following system of differential equations for problems 1 and 2:

$$\frac{dx}{dt} = y + y^2$$
$$\frac{dy}{dt} = -\frac{x}{2} + \frac{y}{5} - xy + \frac{6y^2}{5}$$

1. Verify that (0,0) is an equilibrium point of the system.

2. Find the other equilibrium point of the system.

3. Give an example of a partially decoupled system of differential equations.

4. Determine whether $(4e^{2t} - e^{-t}, 3e^{-t})$ is a solution to the system of differential equations

$$\frac{dx}{dt} = 2x + y$$

$$\frac{dy}{dt} = -y.$$

For questions 5 through 7, consider a situation where the populations of felines and rodents in a region follow the differential equations

$$\frac{dF}{dt} = -5F + 0.01FR$$

$$\frac{dR}{dt} = 3R - 0.2FR.$$

5. Use Euler's Method with a step size of $\Delta t = 0.25$ to estimate the populations of felines and rodents in the region after 1 month if those populations begin at F = 10 and R = 400. [Keep all of your intermediate results to at least the nearest tenth of a creature, however silly that might be.]

6. Use Euler's Method with a step size of $\Delta t = 1$ to estimate the populations of felines and rodents in the region after 1 month with the initial condition above. Explain how valid this approximation is likely to be and why.

7. Explain what the positive and negative signs on the FR terms in the system of differential equations on the previous page mean, that is, what do they tell you about the situation being represented?

8. Suppose the functions $y = e^{2t}\cos(10t)$, $y = e^{-0.2t}\cos(10t)$, $y = e^{-2t}\sin(10t)$, and $y = e^{0.2t}\sin(10t)$ are proposed as possible solutions to a system of differential equations representing the position of a spring acting in a resisting medium. Assess each of these candidate solutions – which of these are particularly reasonable or unreasonable possibilities, and why?

9. Consider the differential equation $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} - y = 2x$. Suppose someone suggests that a solution of the form $y = Ax^2 + Bx + C$ exists, for some values of A, B, and C. Find appropriate values of A, B, and C, if there are any.

10. Solve the system of differential equations

$$\frac{dx}{dt} = -\frac{1}{2}x + 3y$$

$$\frac{dy}{dt} = x.$$