**Exam 3 Differential Equations 5/7/2003**

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. If you know that $\lambda_1 = -3$ and $\lambda_2 = -2$ are eigenvalues of the coefficient matrix of a planar linear system, and that $\mathbf{V}_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\mathbf{V}_2 = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$ are the corresponding eigenvectors, write a **general solution** to the system.

2. Give an **example** of a coefficient matrix for a planar linear system in which the origin would be classified as a saddle.

3. Give an **example** of a system of two linear differential equations (with real coefficients) whose eigenvalues will be complex.

4. **Sketch the phase portrait** for a system of planar differential equations where $\lambda_1 = -2$ and $\lambda_2 = 1$, with $\mathbf{V}_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ and $\mathbf{V}_2 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$ being the corresponding eigenvectors.

5. Given that the planar system

$$\frac{d\mathbf{Y}}{dt} = \begin{pmatrix} -4 & 1 \\ 2 & -3 \end{pmatrix}\mathbf{Y}$$

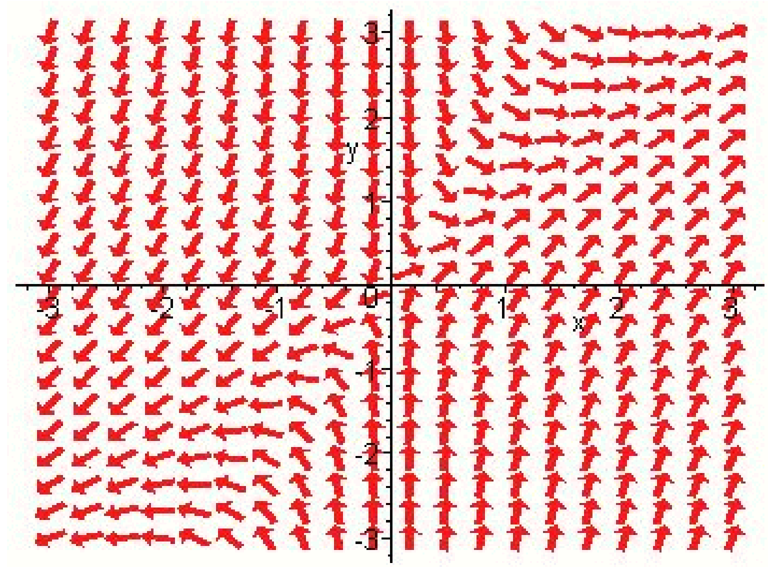
has general solution

$$\mathbf{Y}(t) = k_1 e^{-5t}\begin{pmatrix} 1 \\ -1 \end{pmatrix} + k_2 e^{2t}\begin{pmatrix} 1 \\ 2 \end{pmatrix},$$

**find the particular solution** satisfying $\mathbf{Y}_0 = (1,0)$.

6. **Give a general solution** to the system of differential equations

$$\frac{dx}{dt} = 2x + 0\,y$$
$$\frac{dy}{dt} = 4x - 3y$$



7. **Give a condition** under which the planar system associated with the differential equation

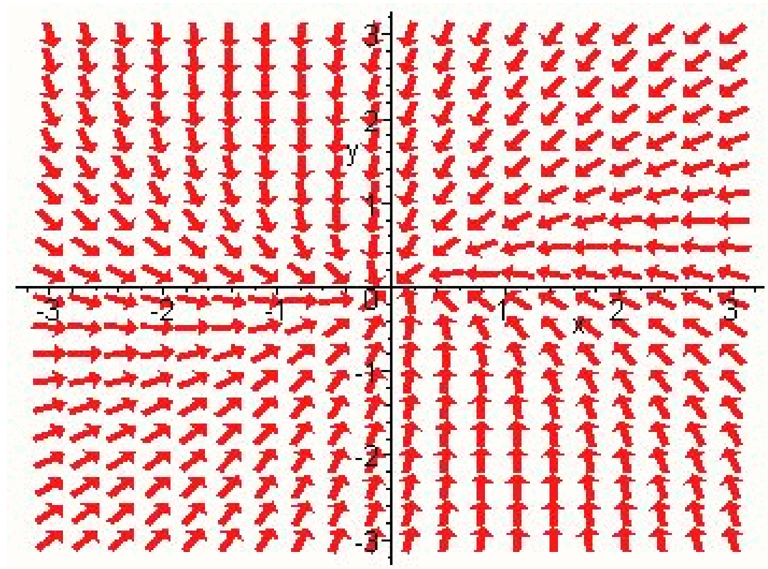
$$ay'' + by' + cy = 0$$

will have purely imaginary eigenvalues.

8. For the planar system

$$\frac{d\mathbf{Y}}{dt} = \begin{pmatrix} -2 & -1 \\ 1 & -4 \end{pmatrix}\mathbf{Y},$$

**Find the particular solution** satisfying the initial condition $\mathbf{Y}_0 = (0,1)$.



9. **Show** that a system of the form

$$\frac{dx}{dt} = ax + by$$

$$\frac{dy}{dt} = -bx + ay$$

with b $\neq$ 0 must have complex eigenvalues.

10. Let

$$\mathbf{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}.$$

Define the trace of **A** to be tr(**A**) = $a + d$. **Show** that A has only one eigenvalue if and only if $(\mathrm{tr}(\mathbf{A}))^2 - 4\det(\mathbf{A}) = 0$.