

$$2.9) \frac{dy}{dt} = 2-y$$

$$\frac{dy}{2-y} = dt$$

$$-\int \frac{1}{u} du = \int dt$$

$$\ln(2-y) = -t + C$$

$$2-y = ke^{-t}$$

$$\underline{2 - ke^{-t} = y}$$

$$\underline{\text{check}}) \frac{dy}{dt} = ke^{-t}$$

$$2 - (2 - ke^{-t}) = ke^{-t} \checkmark$$

$$u = 2-y$$

$$-du = dy$$

Great

② #2, section 1.6

Sketch the phase lines for the given differential equation.

Identify the equilibrium points as sinks, sources, or nodes.

$$\frac{dy}{dt} = y^2 - 6y - 16 = 0$$

$$(y-8)(y+2) = 0$$

$$y = -2, 8$$

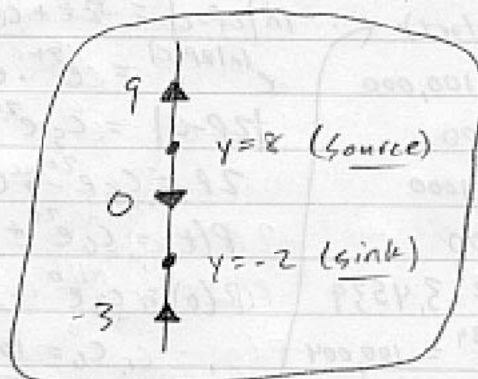
$$f(-3) = (-3)^2 - 6(-3) - 16$$

$$9 + 18 - 16 > 0$$

$$f(0) = 0 - 0 - 16 < 0$$

$$f(9) = 9^2 - 6 \cdot 9 - 16$$

$$81 - 54 - 16 = 11 > 0$$



Good

Section 1.7 - #8:

$$\frac{dP}{dt} = kP - C$$

$$P_0 = 100$$

$$k = 2$$

$$0 = kP - C$$

When $kP = C$, it will be in equilibrium.

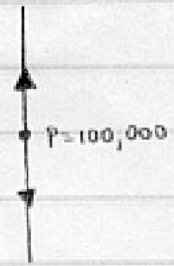
$$2(100,000) = C$$

$$C = 200,000$$

C , the harvesting rate, needs to be set at 200,000. Yes, we can use this particular rate forever as long as the population and growth-rate coefficient stay the same. If one of these changes, then your harvesting rate will have to change also.

The population needs to be monitored rather closely, because if it starts to deviate, it will rapidly go away from your equilibrium because the point is a source and the equilibrium is unstable.

Exactly



(4) Rate at which salt enters per minute = 2
 " " " " leaves " " = $\frac{S}{15+t}$

Volume at a time $t = 15 + (2-1)t$

\therefore Time it takes to fill tank, $15 + t = 30$
 $\Rightarrow t = 15$ mins.

$$\therefore \frac{dS}{dt} = 2 - \frac{S}{15+t}$$

$$\Rightarrow \frac{dS}{dt} + \frac{1}{15+t} \cdot S = 2$$

$$\Rightarrow (15+t) \cdot \frac{dS}{dt} + \frac{15+t}{15+t} \cdot S = 2 \cdot (15+t)$$

$$\Rightarrow \frac{d}{dt} (S \cdot (15+t)) = 2t + 30$$

$$\Rightarrow \underline{S(15+t) = t^2 + 30t + C}$$

At, $t = 0$, $S = 6$

$$\Rightarrow 6(15+0) = 0 + C$$

$$\Rightarrow \underline{C = 90}$$

$$\therefore S = \frac{t^2 + 30t + 90}{t+15}$$

\therefore At $t = 15$,

$$S = \frac{15^2 + 30 \cdot 15 + 90}{15+15} = \underline{25.5 \text{ pounds}} \quad \boxed{\text{Ans}}$$

Well done



$$\begin{aligned} \mu &= 2 \int \frac{1}{15+t} dt \\ &= 2 \ln|15+t| \\ &= \underline{15+t} \end{aligned}$$