Problem Set 2 Differential Equations Due 4/2/2003

Each problem is worth 5 points. For full credit provide complete justification for your answers.

1. Do problem #14 in section 2.3.

2. We discussed in class a differential equation model for a fountain with three chambers when a contaminant is introduced into one of the chambers. Pick some reasonable parameters and use Euler's method to investigate such a system. Discuss how what you find meets or differs from your expectations.

3. On the island of Komodo there live three species: reptiles, mammals, and plants[1]. We will represent the numbers of these species, in thousands, as R, M, and P respectively. The following differential equations give the rates of growth (where t is in years) of reptiles, mammals, and plants on the island:

$$\frac{dR}{dt} = -0.2R - 0.04RM + 0.0008RP$$

$$\frac{dM}{dt} = -0.1M + 0.01RM$$

$$\frac{dP}{dt} = 2P - 0.002P^2 - 0.1RP$$

a) If the populations start at R = 6, M = 3, and P = 600, use Euler's method (with $\Delta t = 1$) to estimate the populations when t = 8.

b) Find the values of R, M, and P for which the Island of Komodo is in equilibrium.

c) Describe what would happen to each of the populations on Komodo if the other two species were not present (i.e., what would happen to the Reptiles if there were no Mammals or Plants, etc.). You may want to use technical terms such as "exponential growth", "logistic growth", or "dying like flies".

d) Who is eating whom on Komodo? Describe the nature of the interaction between each class.

4. The Mammals on Komodo begin drinking a can of Mountain Dew every hour, which makes them much fiercer predators but without the attention span to finish a meal. This changes the differential equation for the reptiles to:

$$\frac{dR}{dt} = -0.2R - 0.08RM + 0.0008RP$$

and leaves the other equations unchanged. What happens to the populations?

[1]The basis for these problems was ripped off from Instructor's Manual with Sample Exams to Accompany Calculus by Hughes-Hallet, Gleason, et el.