Problem Set 5 Differential Equations Due 5/18/2003

Each problem is worth 5 points. For full credit provide complete justification for your answers.

1. Find a series solution of at least 5th degree to the differential equation $y'' + y = 0$ and satisfying the initial condition y(0) = 1.

2. Find a series solution of at least 5th degree to the differential equation $(x-3)y' + 2y = 0$ satisfying the initial condition $y(0) = c_0$.

3. Find a series solution of at least 5th degree to the differential equation $x^2y' + y = 0$ satisfying the initial condition y(0) = 1.

4. Find a general solution to the "fountain system" $\begin{aligned} \frac{dx}{dt} &= -0.1x + 0.1y \\ \frac{dy}{dt} &= \quad 0.1x - 0.1y \end{aligned}$ and sketch its phase plane.

5. Find a general solution to the system $\begin{aligned} \frac{dx}{dt} &= \quad 2y \\ \frac{dy}{dt} &= -x + 2y \end{aligned}$ and classify the system as a source, sink, etc.

6. Find a general solution to the differential equation $\frac{dy}{dt} = 2t - 5$.

7. Find a general solution to the differential equation $\left(1 + t^2\right)\frac{dy}{dt} - 2ty - 2 = 0$.

8. Make up a planar system that's not totally trivial and solve it, then classify it as a source, sink, etc.