Exam 1a **Calc 2** **1/30/2004**

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. Use a table to **integrate** $\int x^2 e^{5x}\, dx$.

$P(x) = x^2$

$a = 5$

$\int x^2 e^{5x} = \frac{1}{5} x^2 e^{5x} - \frac{1}{25} \cdot 2x\, e^{5x} + \frac{1}{125} \cdot 2 \cdot e^{5x}$

$= \underline{\frac{1}{5} x^2 e^{5x}} - \underline{\frac{2}{25} x e^{5x}} + \underline{\frac{2}{125} e^{5x}} + \underline{C}$

$\underline{\text{nt} - 1}$

Good

2. If an ugli fruit is thrown upward at 30 feet per second from a height of 12 feet [and acceleration due to gravity is 32 feet per second^2 downward], find formulas for the egg's velocity and height after t seconds.

$a = -32\ ft/s^2$

$\therefore v = -32t + C$, when $t = 0$,

$v = -32 \cdot 0 + C = 30$

$\boxed{\therefore \frac{ds}{dt} = v = -32t + 30}$

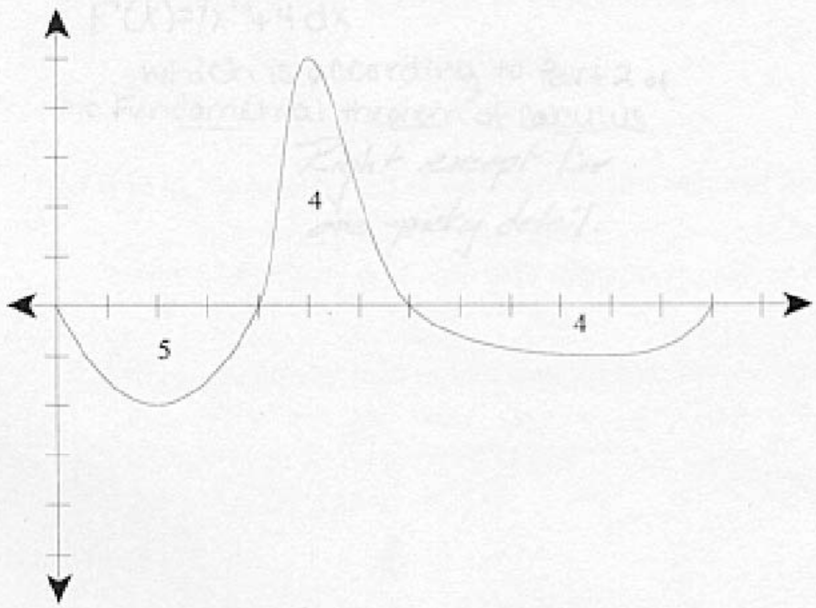
Great

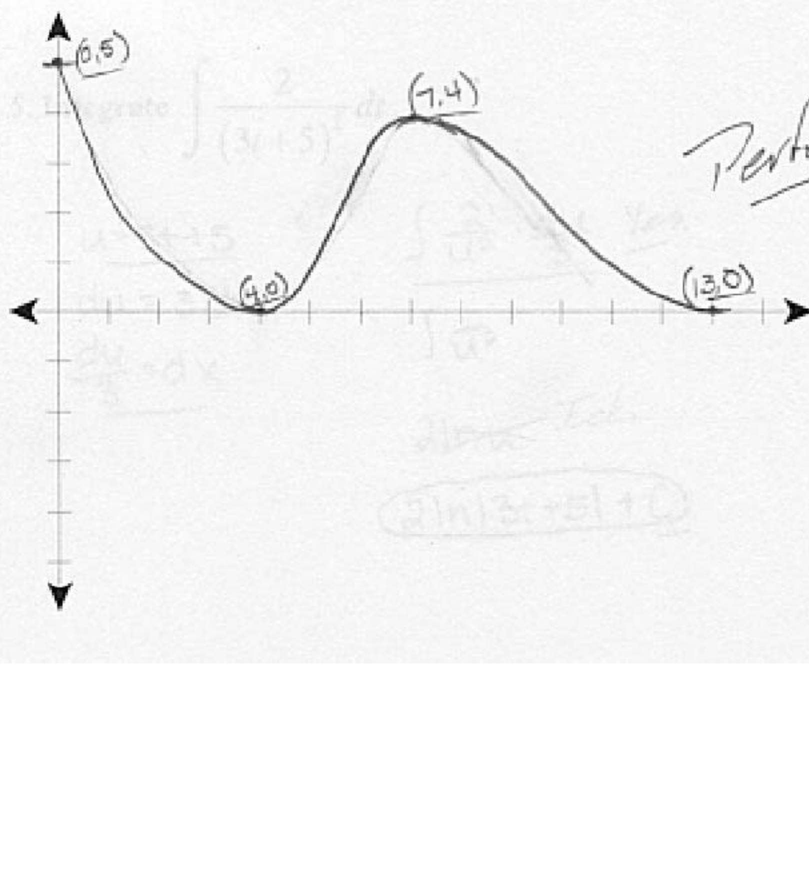
$\therefore S = -16t^2 + 30t + C$, when $t = 0$,

$S = 0 + 0 + 12$

$\therefore \boxed{S = -16t^2 + 30t + 12}$

3. Given the graph of $F'(x)$ shown below (with the areas of several regions marked) and the fact that $F(0) = 5$, sketch the graph of $F(x)$ and label the coordinates of all critical points on the graph of $F(x)$.





Perfect

4. If $F(x) = \int_x^1 \sqrt{t^6+4}\,dt$, what is $F'(x)$?

$= -\sqrt{x^6+4}$

it becomes negative to switch the integral around, x shouldn't be on the bottom x replaces t by deriving the antiderivative

Exactly.

5. Integrate $\int \frac{2}{(3t+5)^2}\,dt$.

let $u = 3t+5$

$\frac{du}{dt} = 3$

$dt = \frac{du}{3}$

$= \int \frac{2}{u^2} \cdot \frac{du}{3}$

$= \frac{2}{3} \int \frac{1}{u^2}\,du$

$= \frac{2}{3} \frac{u^{-2+1}}{-2+1} + C$

$= \frac{2}{3} \frac{u^{-1}}{-1} + C$

$= -\frac{2}{3} \frac{1}{u} + C$

$= -\frac{2}{3} \frac{1}{(3t+5)} + C$

Nice!

6. If $\int_1^3 e^{(-x^2)}$ has been approximated with $L_{20} = 0.1583$, $R_{20} = 0.1216$, and $M_{20} = 0.1391$,

(a) What are T_{20} and S_{20} (rounded to 4 decimal places)?

$T_{20} = \frac{L_{20} + R_{20}}{2} = \frac{.1583 + .1216}{2}$ $\boxed{T_{20} = .1400}$

$S_{20} = \frac{2M_{20} + T}{3} = \frac{2(.1391) + .1400}{3}$ $\boxed{S_{20} = .1394}$

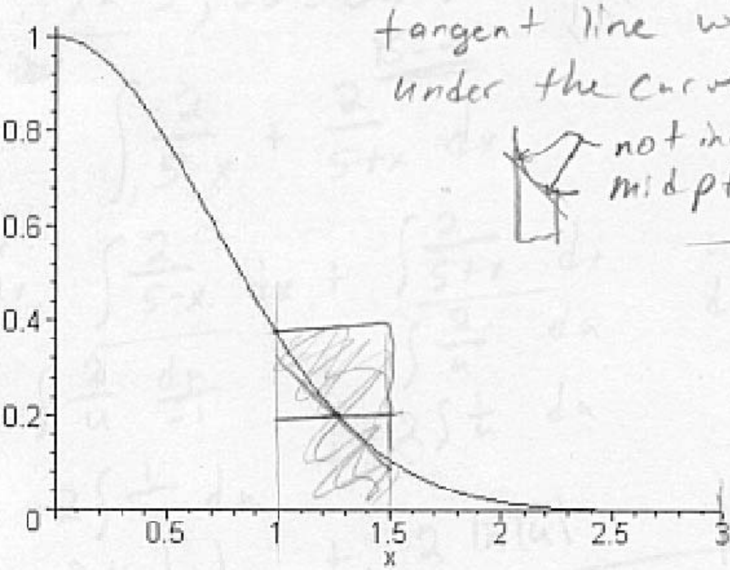
(b) Will L_{20} be greater than or less than the true value of the integral? How can you tell?

Greater; since the graph is sloping down from left to right, estimating the area from the left sum will include area not included on the actual integral

(c) Will M_{20} be greater than or less than the true value of the integral? How can you tell?

less than; Using the Midpoint estimation is like making a trapezoid where one side is formed from the tangent to the curve. Since this fcn is concave up, that tangent line will not include two tiny pieces under the curve and be an underestimate.

not included in midpt estimation.



Excellent

$(5+x)(5-x)$

$-25-5x+5x-x^2$

7. Use partial fractions to reduce $\int \frac{20}{25-x^2}\,dx$ to two simpler integrals and integrate them.

$\int \frac{20}{(5+x)(5-x)}dx \Rightarrow 20 = \frac{A}{(5+x)} + \frac{B}{(5-x)}$

$20 = A(5-x) + B(5+x)$

let $x=5$; $20 = A(5-(5)) + B(5+(5))$

$\frac{20}{10} = \frac{10}{10}B$

$\boxed{B=2}$

let $x=-5$; $20 = A(5-(-5)) + B(5+-5)$

$20 = \frac{10}{10}A$

$A=2$

10 Excellent

$\int \frac{2}{5+x}dx + \int \frac{2}{5-x}dx$

$\int \frac{2}{5+x}dx$ let $u=5+x$, $\frac{du}{dx}=1$, $du=dx$

$\int \frac{2}{u}du$

$2\int \frac{1}{u}du$

$2\cdot \ln|u|$

$2\cdot \ln|5+x|+C$

$\int \frac{2}{5-x}dx$ let $u=5-x$, $\frac{du}{dx}=-1$, $-du=dx$

$\int \frac{2}{u}\cdot\frac{du}{-1}$

$-2\int \frac{1}{u}du$

$-2\cdot \ln|u|$

$-2\cdot \ln|5-x|+C$

$\boxed{2\cdot \ln|5+x| - 2\ln|5-x| + C}$

8. Biff is a calculus student at Factory State University, and he's having some trouble. Biff says "Dude, I totally don't understand this intergation stuff. I can't understand my teacher because he's got so much accent and besides he only faces the chalkboard the whole class anyway. But I take really good notes, even if I don't know what they mean, so I've caught on that you put a "+C" at the end of a lot of problems. I guess it must be important, but I've got no clue what it means. Is it like some famous math guy's initial or something?"

Explain clearly to Biff **when** an answer involves a "+C" and **why**.

The answer involves a "+C" only in an indefinite integral. This is because when you take the derivative of a function, all constants are 0 because you multiply the term by n (where n is the exponent on the variable). You can write a constant (say 5) as $5x^0$ because anything to the 0 power is 1; so when you take the derivative, it's 0. Because of this, if there are no set limits to the integral, you can't determine if there was a constant added to the original function or not, and you can't tell what it is. So, we add the generic "+C" to the end. ~~so~~

Excellent

9. Find a formula for $\int_0^1 e^{ax}\,dx$ in terms of the constant a.

let $\underline{ax = u}$

$$\frac{du}{dx} = a$$

$$dx = \frac{du}{a}$$

$$\Rightarrow \int_{x=0}^{x=1} e^u \cdot \frac{du}{a}$$

$$\Rightarrow \frac{1}{a}\int_{x=0}^{x=1} e^u \, du$$

$$\Rightarrow \frac{1}{a}\cdot e^u \Big|_{x=0}^{x=1}$$

$$\Rightarrow \frac{1}{a}\cdot e^{ax} \Big|_{x=0}^{x=1}$$

$$\Rightarrow \frac{1}{a}\cdot\left(e^a - 1\right)$$

Well done

10. Derive line 24 of our table of integrals, that is, use the trig substitution $x = a\tan\theta$ to show how the integral $\int \frac{1}{x^2+a^2}\,dx$ works out to be $\frac{1}{a}\arctan\frac{x}{a} + C$ [as long as a isn't zero].

10

$\int \frac{1}{x^2+a^2}\,dx$

let $x = a\tan\theta$

$\frac{dx}{d\theta} = a\,\frac{1}{\cos^2\theta}$

$dx = d\theta \cdot a\,\frac{1}{\cos^2\theta}$

$\int \frac{1}{(a\tan\theta)^2+a^2}\,d\theta \cdot \frac{1}{a\cdot\cos^2\theta}$

$\int \frac{1}{a^2\tan^2\theta + a^2}\cdot d\theta\,\frac{1}{a\cdot\cos^2\theta}$

$\frac{1}{a^2}\int \frac{1}{\tan^2\theta+1}\,d\theta\,\frac{1}{a\cos^2\theta}$

$\frac{1}{a^2}\int \frac{1}{\frac{\sin^2\theta}{\cos^2\theta}+\frac{\cos^2\theta}{\cos^2\theta}}$

$\frac{1}{a^2}\int \frac{1}{\frac{\sin^2\theta+\cos^2\theta}{\cos^2\theta}}$

$\frac{1}{a^2}\int \frac{\cos^2\theta}{1}\,d\theta\,\frac{1}{a\cos^2\theta}$

$\frac{1}{a^2}\int 1\cdot\frac{d\theta}{a}$

$\frac{a}{a^2}$

$\frac{1}{a}\int 1\,d\theta$

$\frac{1}{a}\cdot\theta$

but $\frac{x}{a} = \frac{a}{a}\tan\theta$

$\frac{x}{a} = \tan\theta$ arct

$\arctan\frac{x}{a} = \theta$

so

$\boxed{\frac{1}{a}\arctan\frac{x}{a}+C}$

it works

✓ Great Job