Exam 3a Calc 2 3/26/2004

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. Find the sum of the series $1-\frac{2}{5}+\frac{4}{25}-\frac{8}{125}+\ldots$

2. Write the series $(x-1)-\frac{(x-1)^2}{2!}+\frac{(x-1)^3}{3!}-\frac{(x-1)^4}{4!}+\frac{(x-1)^5}{5!}-\ldots$ in sigma notation.

3. Write a 7^{th} degree Taylor polynomial for the function $f(x) = x^2 \sin(2x)$ centered at zero.

4. Several derivatives of the function $f(x) = \sec x$ are given below. Use them to find the 3^{rd} degree Taylor polynomial for $\sec x$ centered at $x = 0$.

$f'(x) = \sec x \tan x$

$f''(x) = \sec x \tan^2 x + \sec x$

$f^{(3)}(x) = \sec x \tan^3 x + 5\sec^3 x \tan x$

5. Determine whether the series $\sum_{n=0}^{\infty} \frac{1}{n^3+1}$ converges or diverges.

6. Is x = -1 included in the interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{x^n}{n}$?

7. What is the radius of convergence of the series $\sum_{n=0}^{\infty}(3x)^n$?

8. Biff is having calculus trouble again. He says "Dude, this series stuff is kicking my ass. I was talking to this friend of mine, and his calculus class uses a different book because he's an engineering major or something, and he was saying there's this other test for whether one converges or not. It's like, you take the square root of it and see if that converges or not, and that tells you if yours converges or not. That seems pretty cool to me, since I was pretty good at math back when it was just doing square roots and stuff."

Tell Biff whether you think he should believe what his friend told him or not, and why. Be clear and justify your answer, keeping it in terms Biff can understand.

9. For which values of p will the series $\sum_{n=0}^{\infty} \frac{e^n}{\left(e^n+1\right)^p}$ converge?

10. Differentiate the Taylor series for xe^x and use the result to show that $\sum_{n=0}^{\infty} \frac{n+1}{n!} = 2e$.

Extra Credit (5 points possible): Solve the equation $1 + x + x^2 + x^3 + x^4 + \ldots = 5$ for x.