Exam 4a Calc 2 4/23/2004

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. Determine whether $y = 3e^{-4t}$ is a solution to the differential equation $\dfrac{d^2 y}{dt^2} = -4y$.

2. Write a differential equation representing a population with logistic growth and a carrying capacity of 500 (use k for the constant of proportionality).

3. Find a general solution to the differential equation $\frac{dy}{dx} + x^2 y^2 = 0$.

4. Find a general solution for the differential equation $y'' - 7y' + 6y = 0$.

5. Find all equilibrium points of the system

$$\frac{dx}{dt} = x - x^2 - \frac{xy}{3}$$

$$\frac{dy}{dt} = y - y^2 - \frac{xy}{2}$$

6. Water leaks from a vertical cylindrical tank through a small hole in its base at a rate proportional to the square root of the volume of water remaining. If the tank initially contains 100 liters and 20 liters leak out during the first day, when will the tank be half empty?

7. Bunny is having trouble with differential equations. She says "Ohmygod, these slopey field thingies are so *hard*. There was this one, and it was, like, one of the system ones, you know? And I did the equations and it said that (0,0) was an equilibrium thingy, but it didn't look like it in the slopey field, 'cause all the arrows pointed away from there, so that's not an equilibrium, right? So I don't know what to do."

Explain to Bunny whether what she's saying is true, and how to tell from a slope field whether a point is an equilibrium.

8. Lake Erie has a volume of 460,000 km^3 and an outflow rate of 175 km^3 per year. Suppose that 20kg of a certain pollutant is present in the lake at time 0, when a massive filtration project is undertaken to remove the substance from the lake. If 0.1kg per year is removed from the lake beyond the amount removed by natural processes,

a) Write a differential equation for the amount of pollutant in the lake after t years.

b) Use Euler's method with $\Delta t = 5$ to estimate the pollutant remaining in the lake after 10 years.

9. Show that the function $y = te^{-t}$ is a solution to any differential equation of the form

$$ay'' + 2ay' + ay = 0$$

regardless of the value of the constant a.

10. Find a general solution to the differential equation $y'' + \omega^2 y = 0$, where ω is a constant.

Extra Credit (5 points possible): Try to find a solution $y(t)$ of the differential equation $y'' - ty' - 2y = 0$.