Practice Exam 4   Calc 2   3/26/2004

Each problem is worth 10 points. For full credit provide complete justification for your answers.

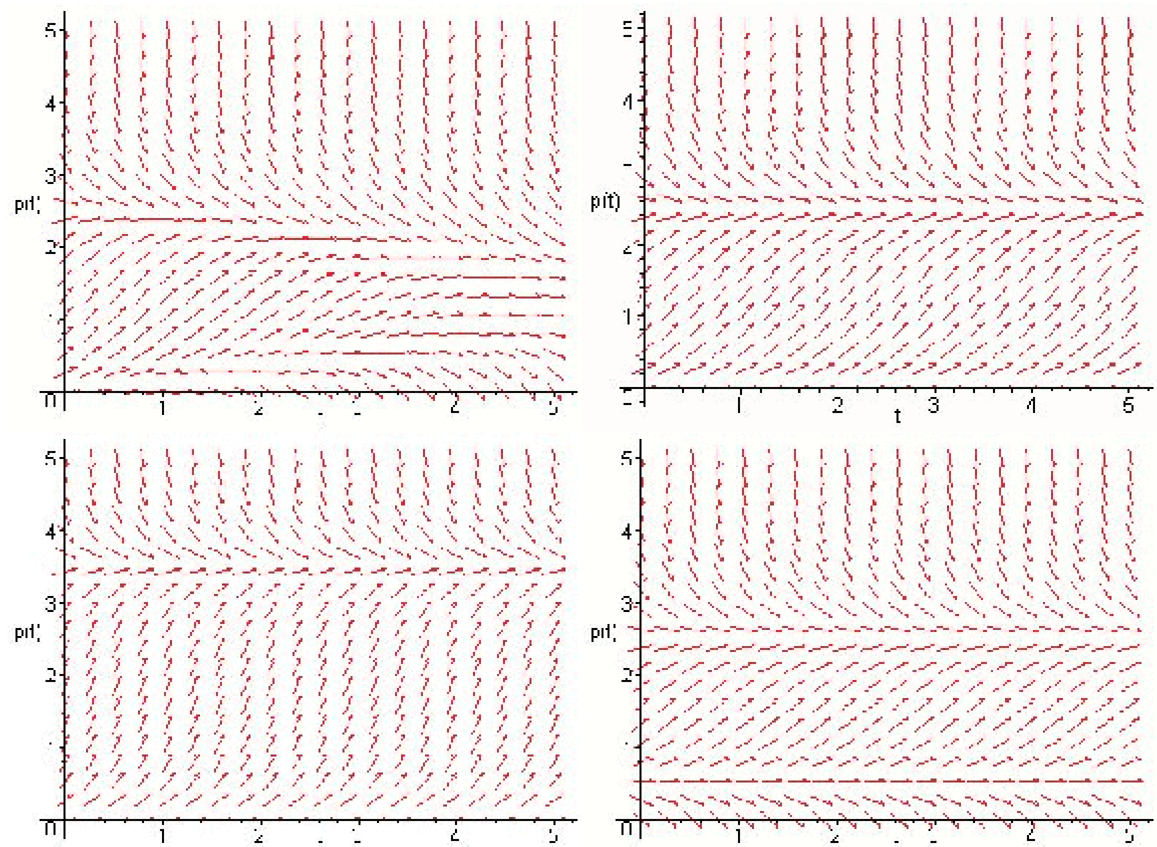
1. Find all equilibrium population values for the differential equation $\frac{dp}{dt} = 0.03p(45 - p)$.

2. a) Find a general solution to the differential equation $\frac{dy}{dx} + xy^2 = 0$.

b) Find a particular solution to the differential equation from part a satisfying the condition $y(1) = 1$.

3. Which of the slope fields pictured below could represent the differential equation

$$\frac{dP}{dt} = -P(P - 2.5)?$$



4. For the differential equation $y'' - 5y' + 6y = 0$,

a) Find a general solution

b) Find the particular solution satisfying the conditions $y(0) = 0$ and $y'(0) = 2$.

5. For the differential equation $y'' + 5y = 0$,

a) Find a general solution

b) Find the particular solution satisfying the conditions $y(0) = 0$ and $y'(0) = 2$.

6. If the differential equation $\frac{dT}{dt} = 0.05(400 - T)$ represents the temperature after t minutes of a toy mouse (which starts out at 70˚) that Jon's cat Nemo drops into the oven, **use Euler's method** with $\Delta t = 5$ to estimate the temperature of the mouse ten minutes later when Jon notices that something smells funny.

7. A yam is removed from an oven and placed in outer space, where the temperature is –250°. If the yam starts out at 300°, and after 5 minutes it's dropped to 100°, how long will it be until it reaches the freezing point, 32°?

8. Biff is having trouble with differential equations. He says "Dude, there was this question on the review for our test, and it was like, whether a solution for $P' = kP(2 - P)$ can go through both (0,1) and (1,3). That's totally screwed up, isn't it? I mean, if they give you the initial things, you can always just work out the constants that make 'em happen, right? How can they even ask that?"

Explain to Biff why this question isn't as trivial as he thinks, and how you would answer it.

9. We've been using the fact that if $f(t)$ and $g(t)$ are solutions to a differential equation of the form $ay'' + by' + cy = 0$ then $f(t) + g(t)$ will also be a solution. Prove it.

10. Consider the differential equation $y'' - 2y' = 2y/t$. It's not quite the form that we can use a characteristic polynomial on, but something similar works: Try to find a solution of the form $y = te^{at}$ for some constant a.