Quiz 2b **Calculus 2** **2/9/04**

Each problem is worth 5 points. For full credit provide proper justification for your answer.

1. Find the value of $\int_0^\infty \frac{x}{e^x}\,dx$ or show that it diverges.

$$= \lim_{b\to\infty} \int_0^b \underbrace{x}_{u} \cdot \underbrace{e^{-x}}_{v'}\,dx$$

let $u = x$ $\quad v = -e^{-x}$

$u' = 1$ $\quad v' = e^{-x}$

$$= \lim_{b\to\infty} \left[x \cdot e^{-x} - \int 1 \cdot -e^{-x}\,dx\right]_0^b$$

$$= \lim_{b\to\infty} \left[\frac{x}{e^x} + (-e^{-x})\right]_0^b$$

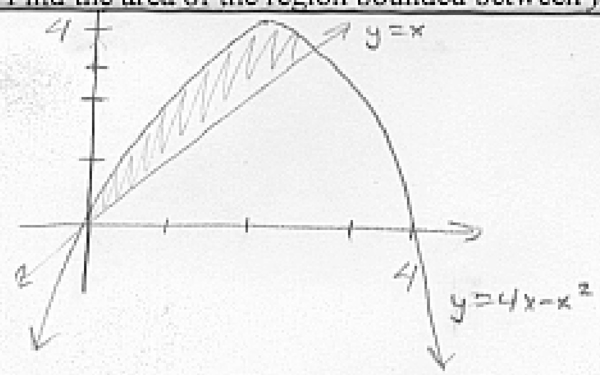
Nice Job!

$$= \lim_{b\to\infty} \left[-\frac{x}{e^x} - \frac{1}{e^x}\right]_0^b$$

$$= \lim_{b\to\infty} -\frac{b}{e^b} - \frac{1}{e^b} + \frac{0}{e^0} + \frac{1}{e^0} \overset{L'H}{=} \lim_{b\to\infty} -\frac{1}{e^b} - 0 + 0 + \frac{1}{1}$$

$= \underline{\underline{1}}$, Ans

2. Find the area of the region bounded between $y = x$ and $y = 4x - x^2$.



$x = 4x - x^2$

$0 = 3x - x^2$

$= x(3 - x)$

$0 = x \quad 3 - x = 0$

$3 = x$

Intersects @ 0 + 3

Nice!

$$\text{Area} = \int_0^3 (4x - x^2)\,dx - \int_0^3 x\,dx$$

$$= \left[2x^2 - \tfrac{1}{3}x^3\right]_0^3 - \left[\tfrac{1}{2}x^2\right]_0^3$$

$$= \left[\left[2(3)^2 - \tfrac{1}{3}(3)^3\right] - \left[2(0)^2 - \tfrac{1}{3}(0)^3\right]\right] - \left[\tfrac{1}{2}(3)^2 - \tfrac{1}{2}(0)^2\right]$$

$$= \left[2(9) - \tfrac{1}{3}(27)\right] - \left[\tfrac{1}{2}(9)\right]$$

$$= 18 - 9 - \tfrac{9}{2}$$

$$= \tfrac{9}{2}$$

Excellent